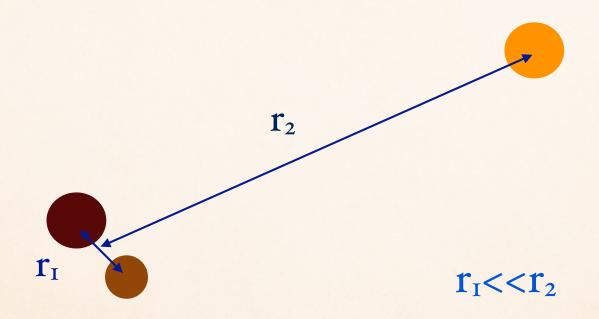


Gongjie Li Harvard University → Georgia Tech

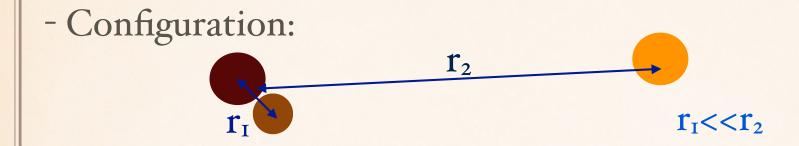
Main Collaborators: Smadar Naoz (UCLA), Bence Kocsis (IAS/Eotvos) Matt Holman (Harvard), Avi Loeb (Harvard)

HIERARCHICAL THREE-BODY SYSTEMS

- Configuration:



HIERARCHICAL THREE-BODY SYSTEMS



- Hierarchical configurations are COMMON:
- For binaries with period < 3 days, ≥96% are in systems with multiplicity ≥3. (*Tokovinin et al. 2006*)
- 282 of the 299 triple systems (- 94.3%) are hierarchical. (Eggleton et al. 2007)
- Hierarchical 3-body dynamics gives insight for hierarchical multiple systems formation/evolution.

OUTLINE

- Dynamical properties:
 - Flips of inner binary
 - Eccentricity excitation of the inner binary
- Examples:
 - Formation of misaligned hot Jupiters
 - Enhancement of tidal disruption rates for stars in galactic nuclei

CONFIGURATION OF HIERARCHICAL 3-BODY SYSTEM

System is stable and can be thought of as interaction between two orbital wires (secular approximation):



CONFIGURATION OF HIERARCHICAL 3-BODY SYSTEM

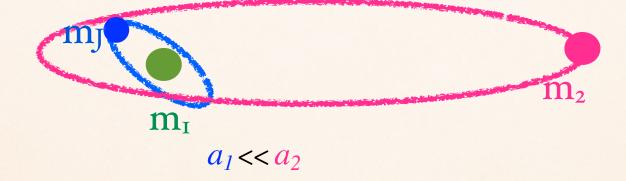
System is stable and can be thought of as interaction between two orbital wires (secular approximation):



- Inner wires (1): formed by m₁ and m_J.
- Outer wires (2): m₂ orbits the center mass of m₁ and m_J.

CONFIGURATION OF HIERARCHICAL 3-BODY SYSTEM

System is stable and can be thought of as interaction between two orbital wires (secular approximation):

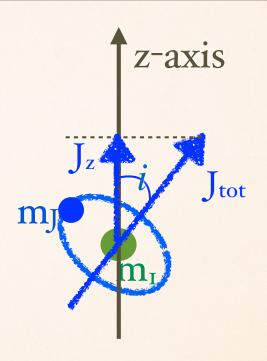


- Inner wires (1): formed by m₁ and m_J.
- Outer wires (2): m₂ orbits the center mass of m₁ and m₁.

Lidov-Kozai Mechanism

- Octupole level $O((a_1/a_2)^3)$ is zero.
- Quadrupole level $O((a_1/a_2)^2)$:

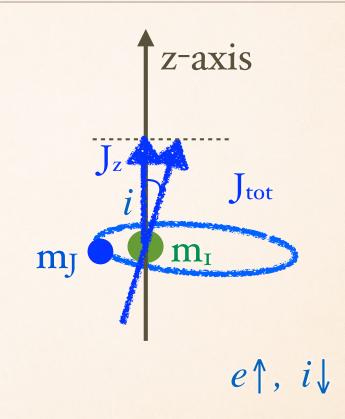
=>
$$Jz = \sqrt{1 - e_1^2} \cos i_1$$
 conserved (axi-symmetric potential).



Lidov-Kozai Mechanism

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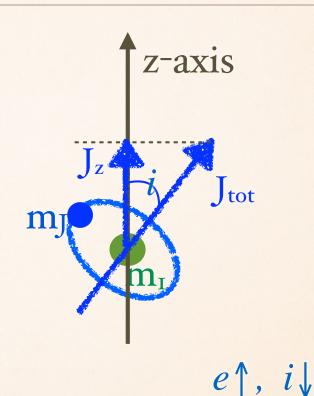
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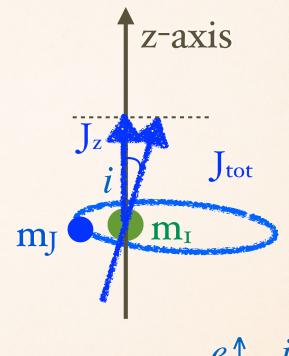
Lidov-Kozai Mechanism

$$(e_2 = 0, m_J \rightarrow 0)$$

 $(Kozai 1962; Lidov 1962:$
 $Solar system objects)$

- Octupole level $O((a_1/a_2)^3)$ is zero.
- Quadrupole level $O((a_1/a_2)^2)$:

=>
$$Jz = \sqrt{1 - e_1^2} \cos i_1$$
 conserved (axi-symmetric potential).



 $e\uparrow$, $i\downarrow$

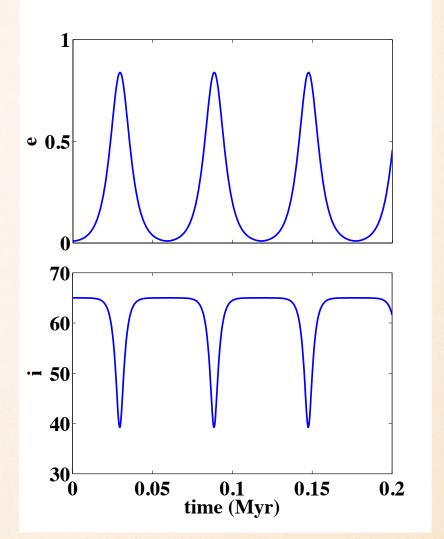
i does not cross 90°

Lidov-Kozai Mechanism

- Octupole level $O((a_1/a_2)^3)$ is zero.
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=>
$$Jz = \sqrt{1 - e_1^2} \cos i_1$$
 conserved (axi-symmetric potential).

=> when i>40°, e₁ and i oscillate with large amplitude.

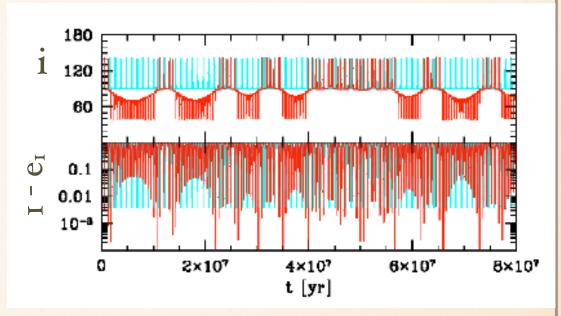


Example of Lidov-Kozai Mechanism.

e₂ ≠ 0 (Eccentric Lidov-Kozai Mechanism) or mJ ≠ 0:

(e.g., Naoz et al. 2011, 2013, test particle case: Katz et al. 2011, Lithwick & Naoz 2011):

- Jz NOT constant, octupole # 0.
- when $i>40^{\circ}$: $e_{I} \to 1$.
- when *i*>40°: *i* crosses 90°



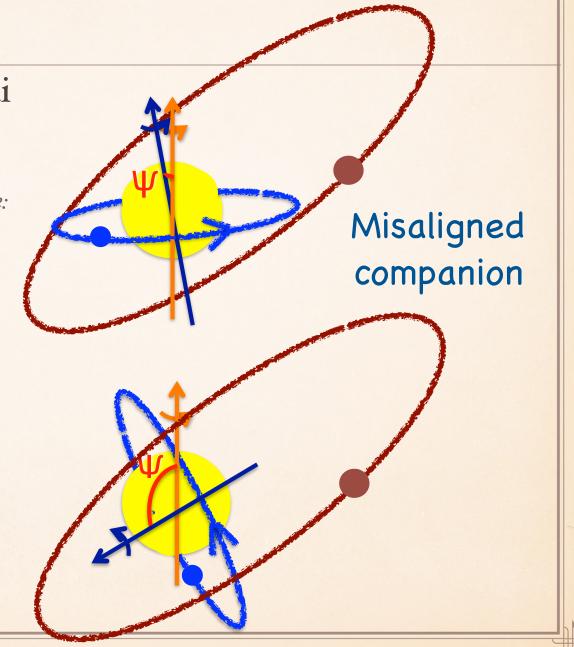
Cyan: quadrupole only.

Red: quadrupole + octupole. Naoz et al 2013

 $e_2 \neq 0$ (Eccentric Lidov-Kozai Mechanism) or $m_J \neq 0$:

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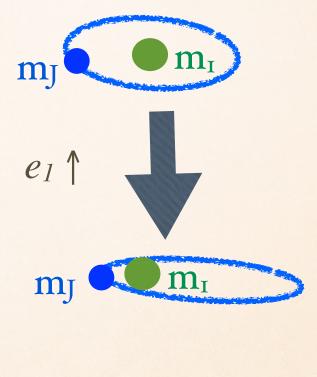
- Consequence:
 - Produces retrograde hot Jupiters (i>90°) (e.g., Naoz et al. 2011)



e₂ ≠ 0 (Eccentric Lidov-Kozai Mechanism) or mJ ≠ 0:

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- Consequence:
 - Tidal disruption rate enhancement (e₁ → 1)
 (e.g., Chen et al. 2009, Bode & Wegg 2014, Li et al. 2015)

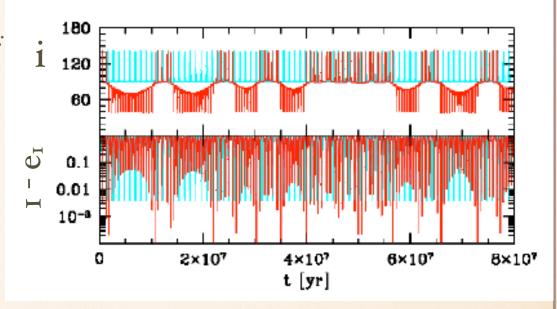


 $R_p \propto 1-e_1$

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- Consequence:
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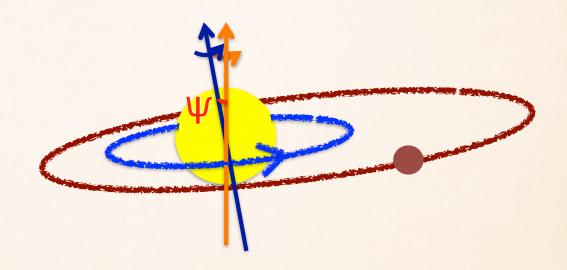


Cyan: quadrupole only.

Red: quadrupole + octupole. Naoz et al 2013

$$40^{\circ} < i < 140^{\circ}$$

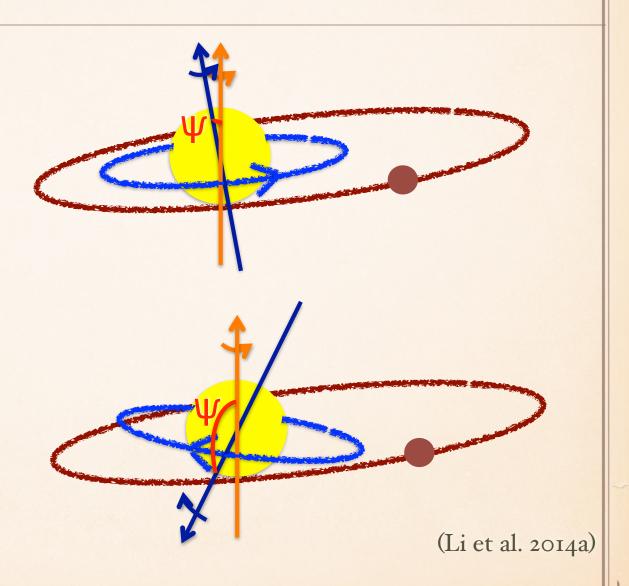
Starting with i ≈ 0,
e₁≥0.6, e₂ ≠ 0:
e₁→1, i flips by ≈180°
(Li et al. 2014a).



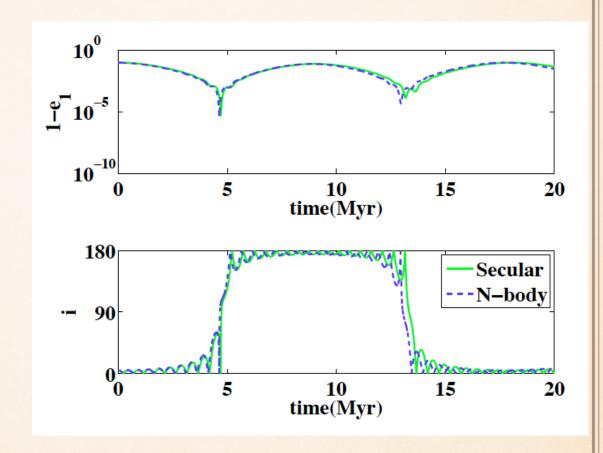
(Li et al. 2014a)

• Starting with $i \approx 0$, $e_1 \ge 0.6$, $e_2 \ne 0$: $e_1 \to 1$, i flips by $\approx 180^\circ$ (*Li et al. 2014a*).



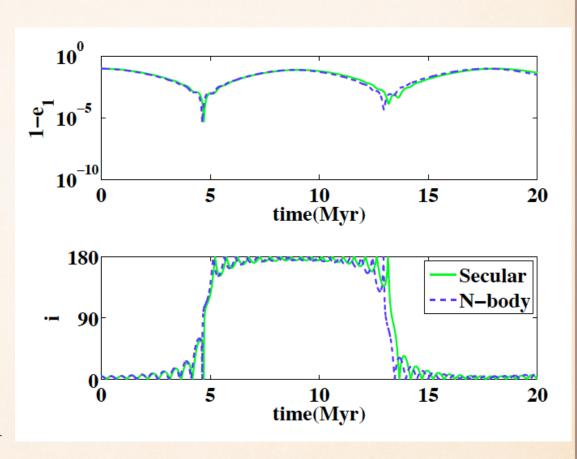


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(Li et al. 2014a)

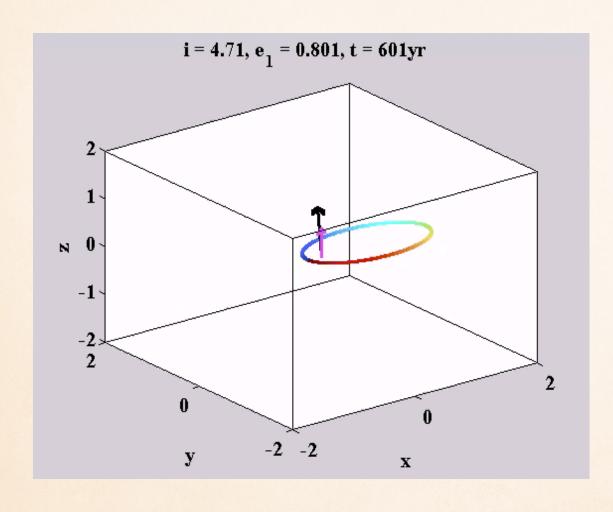
- Starting with i ≈ 0,
 e₁≥0.6, e₂ ≠ 0:
 e₁→1, i flips by ≈180°
 (Li et al. 2014a).
 - => Increase the parameter space of interesting behaviors.
 - => Produces counter orbiting hot Jupiters.
 - => Enhance tidal disruption rates.



(Li et al. 2014a)

DIFFERENCES BETWEEN HIGH/LOW I FLIP

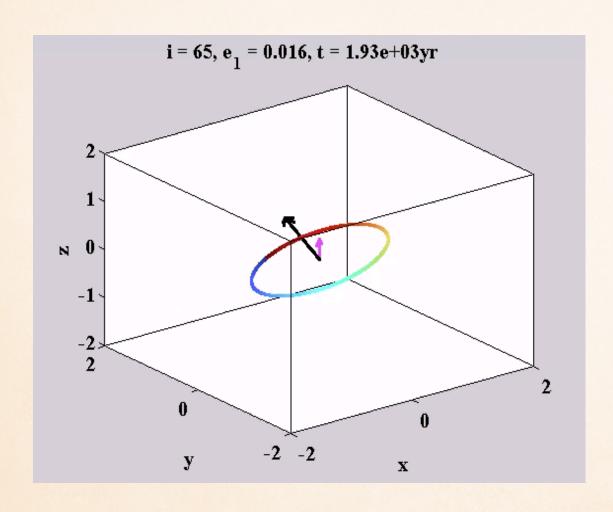
Low inclination flip



- For simplicity: take m_j → o => outer orbit stationary.
- z direction: angular momentum of the outer orbit.
- 1: direction of J₁.
- \uparrow : $Jz_1 => indicates flip.$
- Colored ring: inner orbit.
 Color: mean anomaly.

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ANALYTICAL OVERVIEW

Hamiltonian has two degrees of freedom in test particle limit:

$$(J = \sqrt{1 - e_1^2}, Jz = \sqrt{1 - e_1^2} \cos i_1, \omega, \Omega)$$

2 conjugate pairs: J & ω , Jz & Ω

The Hamiltonian up to the Octupole order:

$$H = F_{quad}(J, Jz, \omega) + \epsilon F_{oct}(J, Jz, \omega, \Omega)$$

Quadrupole order: Independent of Ω => Jz constant ϵ : hierarchical parameter:

$$\epsilon = \frac{a_1}{a_2} \frac{e_2}{1 - e_2^2}$$

Octupole order: Depend on both $\Omega \& \omega \Rightarrow J$ and Jz not constant

CO-PLANAR FLIP CRITERION

- Hamiltonian (at O(i)):
 - Evolution of e_1 only due to octupole terms:
 - $=> e_1$ does not oscillate before flip
 - Depend on only J_{I} and $\varpi_{I} = \omega_{I} + \Omega_{I}$
 - => System is integrable.
 - $=>e_1(t)$ can be solved.
 - => The flip timescale can be derived.
 - => The flip criterion can be derived.

$$\varepsilon > \frac{8}{5} \frac{1 - e_1^2}{7 - e_1 (4 + 3e_1^2) \cos(\omega_1 + \Omega_1)}$$

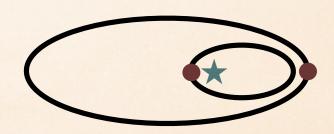
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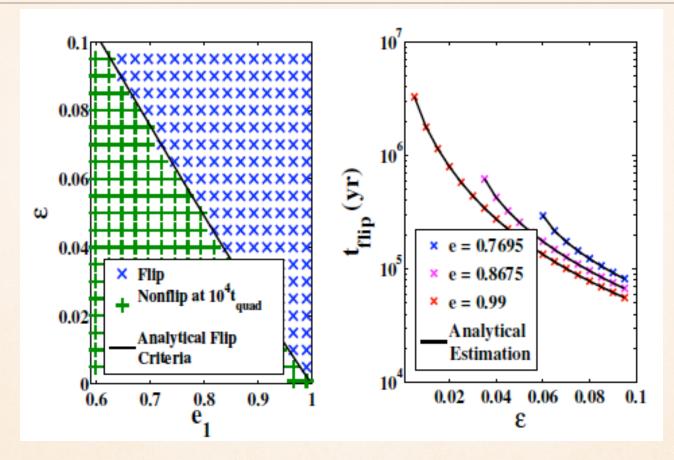
Easier to flip:

* e₁ larger

* $\varpi_1 = \omega_1 + \Omega_1 \sim 180^{\circ}$



ANALYTICAL RESULTS V.S. NUMERICAL RESULTS



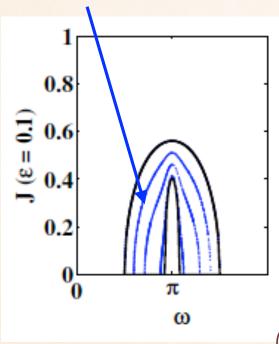
IC: $i=5^{\circ}$.

• The flip criterion and the flip timescale from secular integration are consistent with the analytical results.

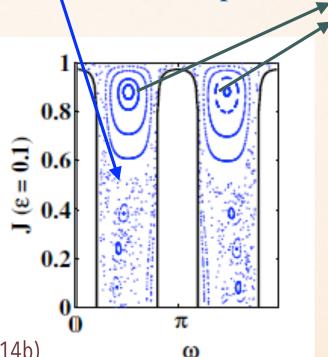
Li et al. 2014a

SURFACE OF SECTIONS





High inclination Flip:



(Gongjie Li et al. 2014b)

Caused by the octupole resonance, Regular (w librates around π)

Caused by the overlap of quadrupole and octupole resonances, Chaotic: t_L-6t_K

Quadrupole

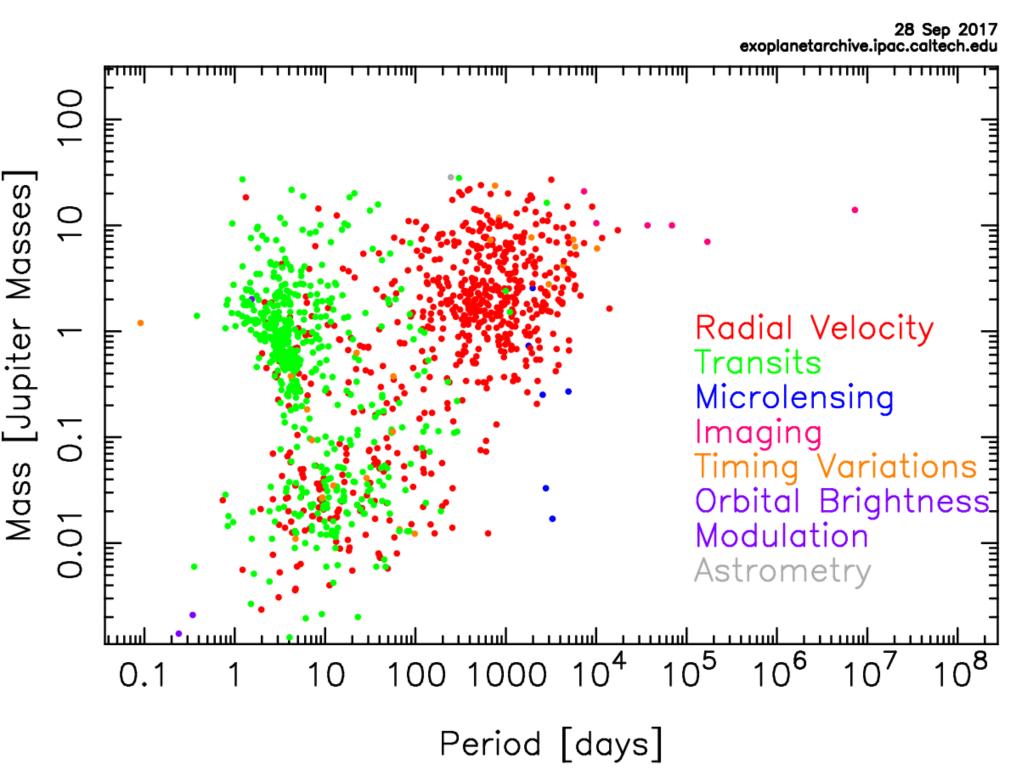
resonances

(e.g., Kozai 1962)

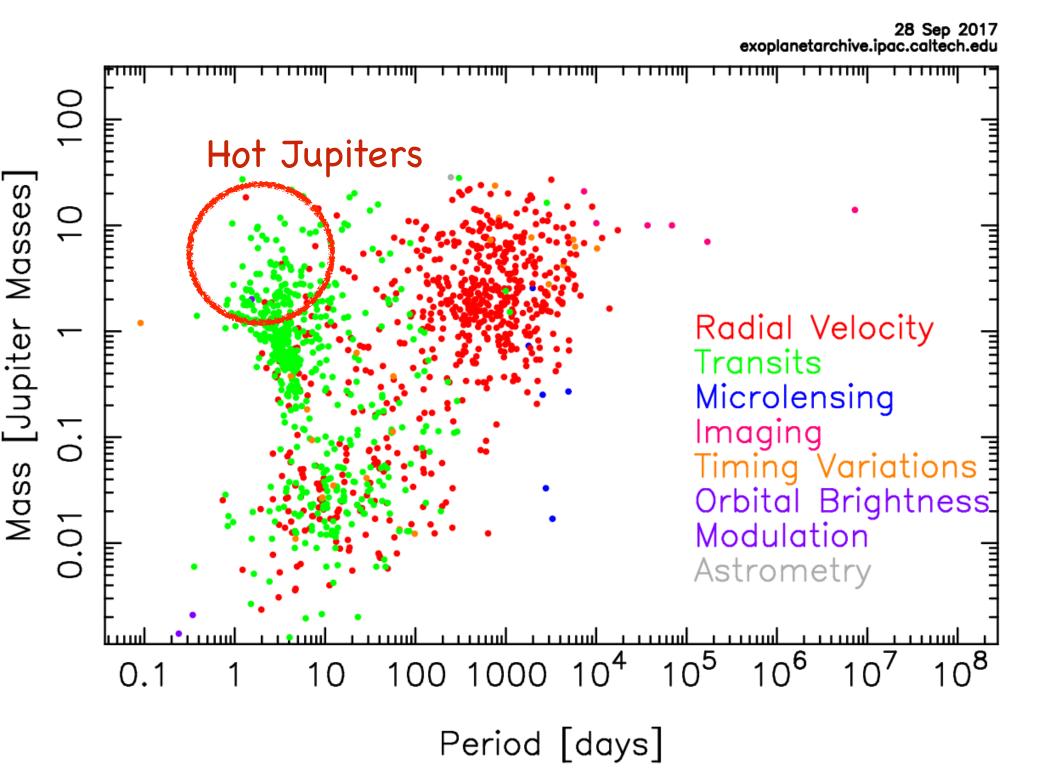
Examples --- 1. Formation of Misaligned Hot Jupiters via Lidov-Kozai Oscillations



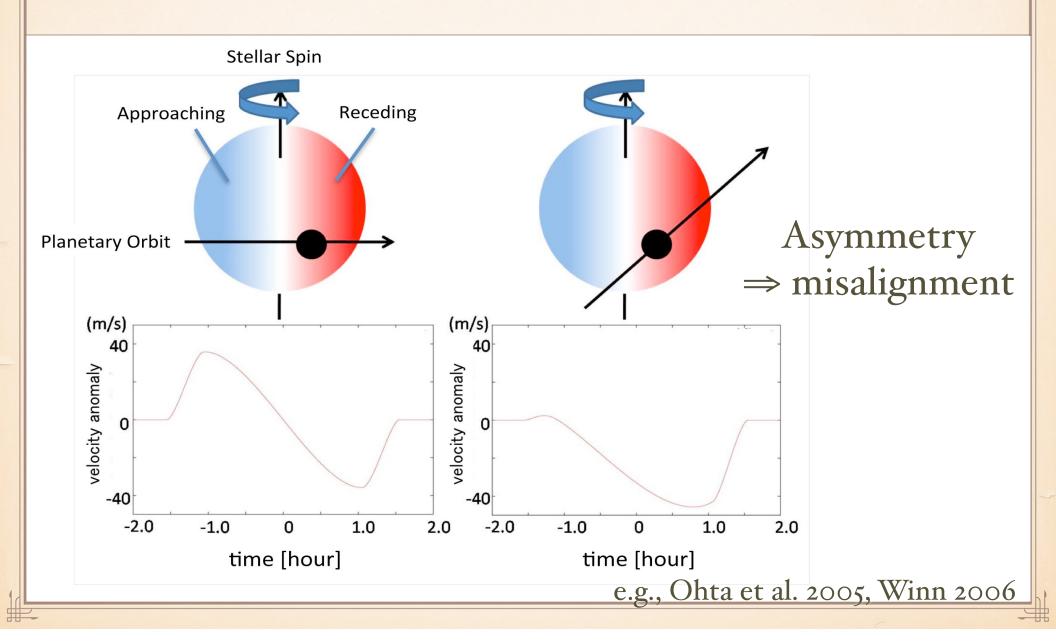
Mass - Period Distribution



Mass - Period Distribution

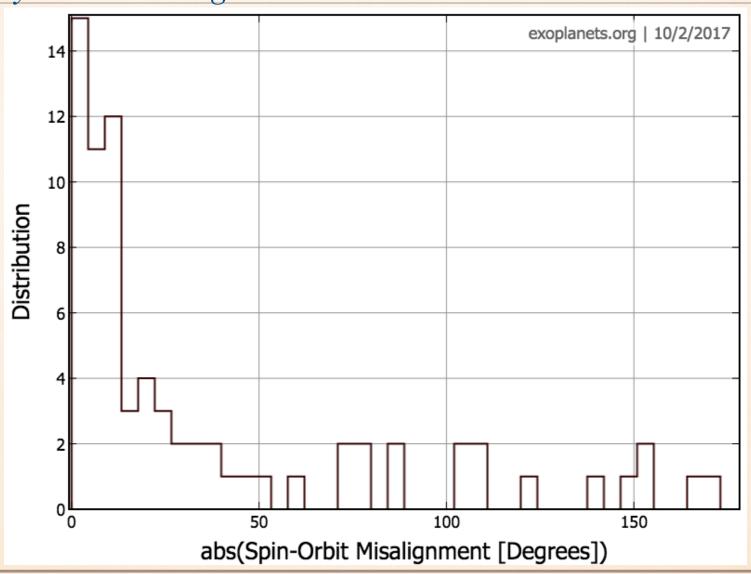


SPIN-ORBIT MISALIGNMENT (ROSSITER-MCLAUGHLIN METHOD)



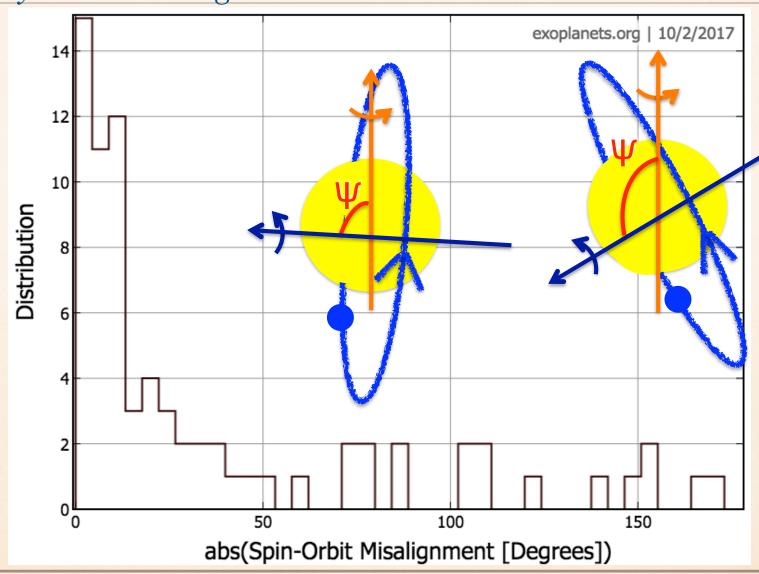
OBSERVED SPIN-ORBIT MISALIGNMENT

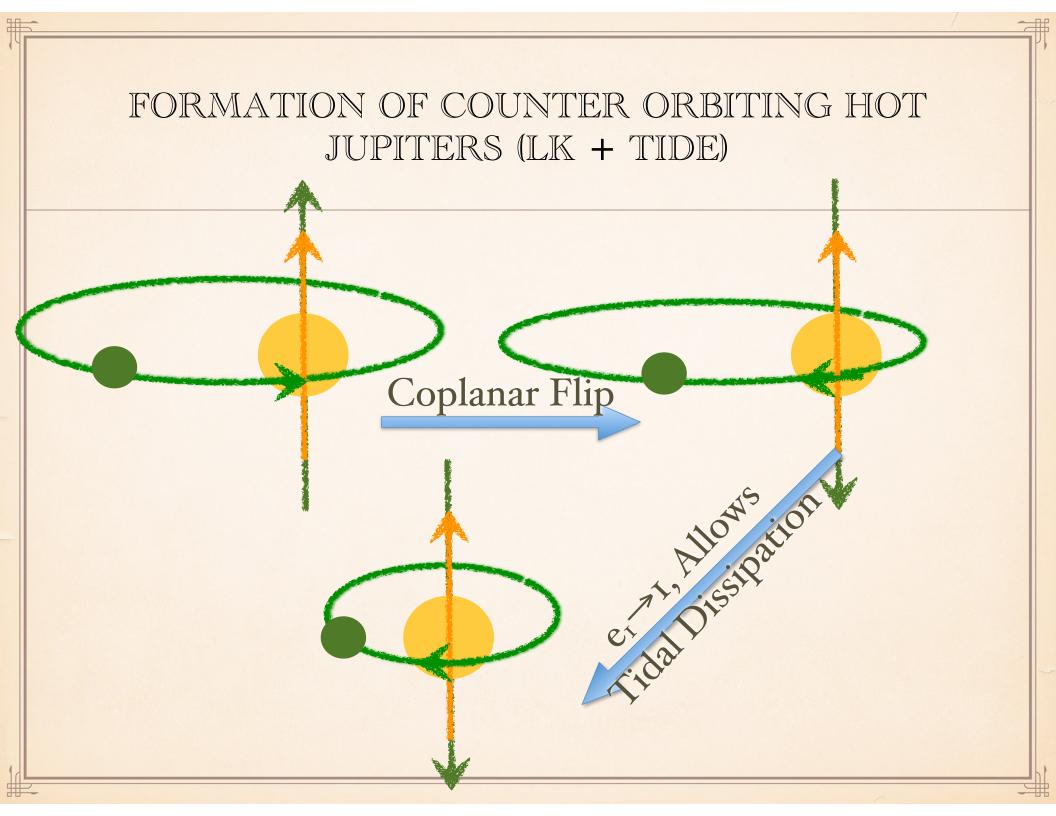
Solar System: misalignment Ψ≤ 7°



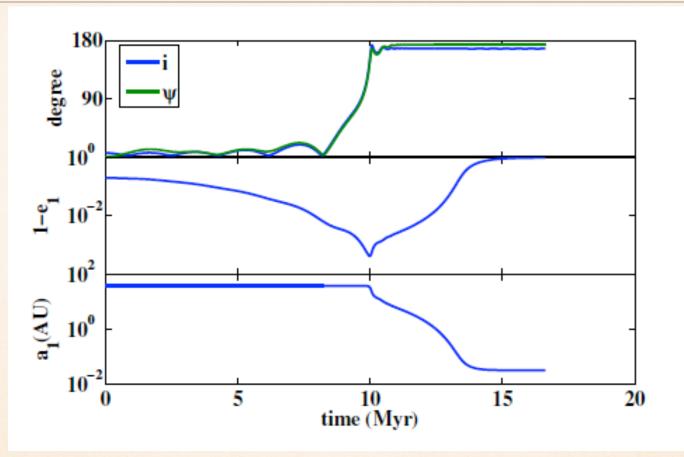
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FORMATION OF COUNTER ORBITING HOT JUPITERS (LK + TIDE)

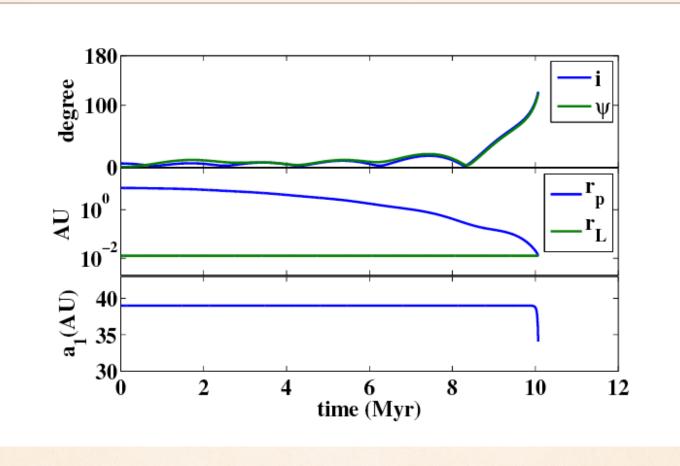


 $e_I \rightarrow 1$ during the flip => $r_p \downarrow$, tide dominates.

$$\Rightarrow e_I \rightarrow 0, a_I \downarrow, i, \psi \approx 180^{\circ}.$$

Li et al. 2014a

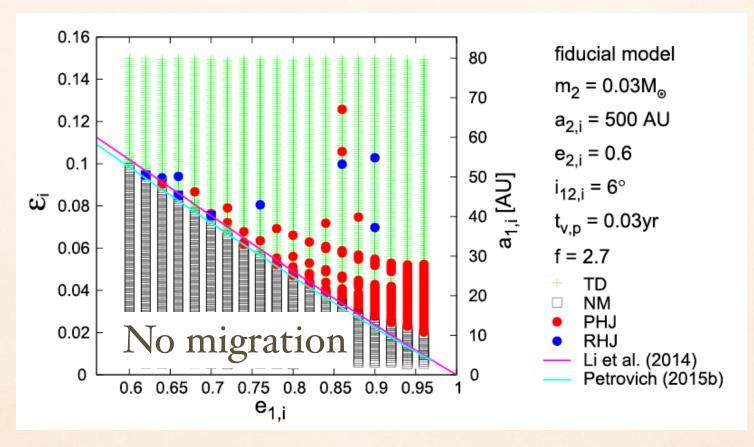
FORMATION OF COUNTER ORBITING HOT JUPITERS (LK + TIDE)



May produce tidal disruption events

DIFFICULTY IN THE FORMATION OF COUNTER-ORBITING HOT JUPITERS

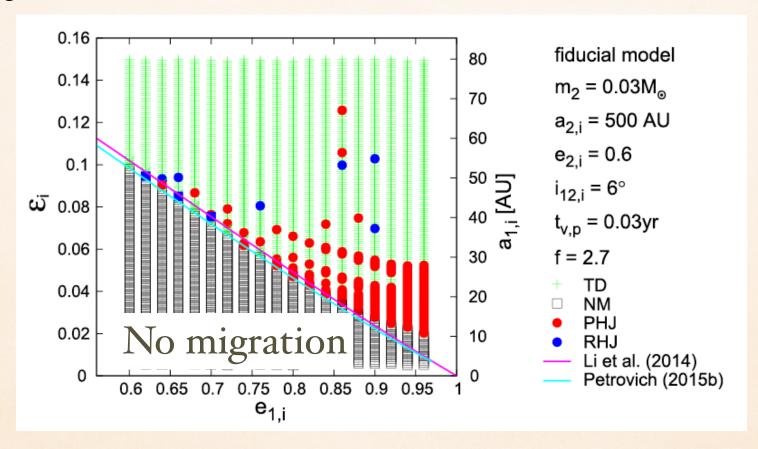
Including short range forces, a small fraction survive and produce retrograde planets



Xue & Suto 2016, Xue et al. 2017

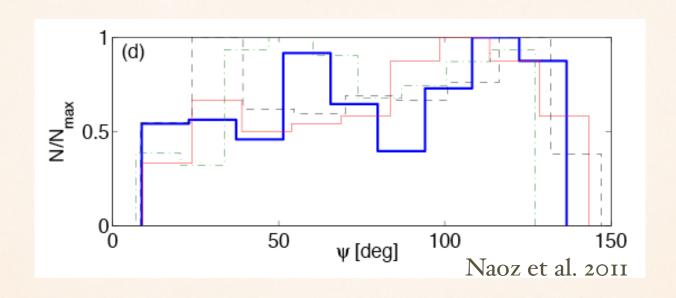
DIFFICULTY IN THE FORMATION OF COUNTER-ORBITING HOT JUPITERS

Flip condition (with no short range forces) is also a good approximation for migration condition



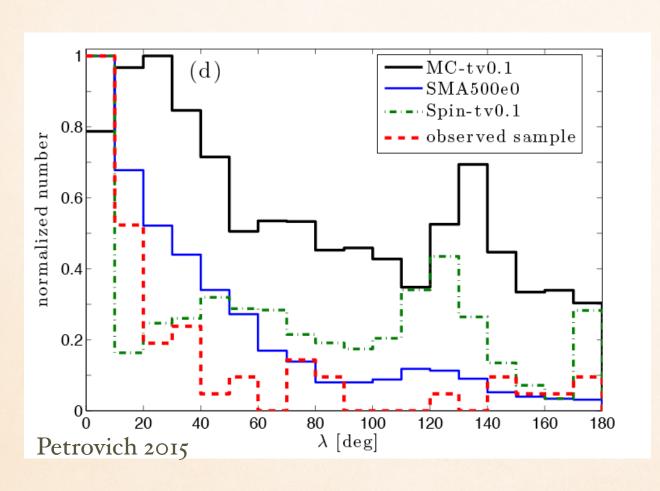
Xue & Suto 2016, Xue et al. 2017

FORMATION OF MISALIGNED HOT JUPITERS (LK + TIDE) BY POPULATION SYNTHESIS



- 15% of systems produce hot Jupiters
- ELK may account for about 30% of hot Jupiters (Naoz et al. 2011)

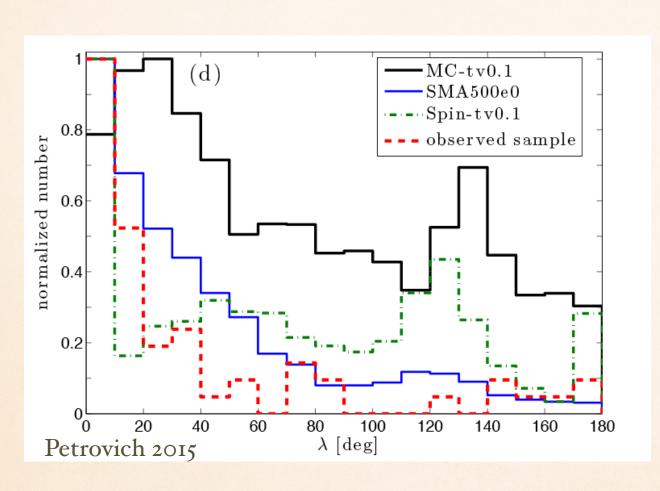
FORMATION OF MISALIGNED HOT JUPITERS (LK + TIDE) BY POPULATION SYNTHESIS



Population synthesis study of interaction of two giant planets.

=> a different mechanism is needed (Petrovich 2015)

FORMATION OF MISALIGNED HOT JUPITERS (LK + TIDE) BY POPULATION SYNTHESIS

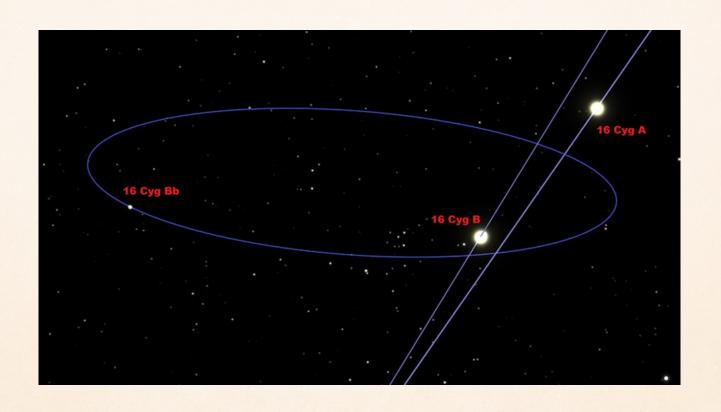


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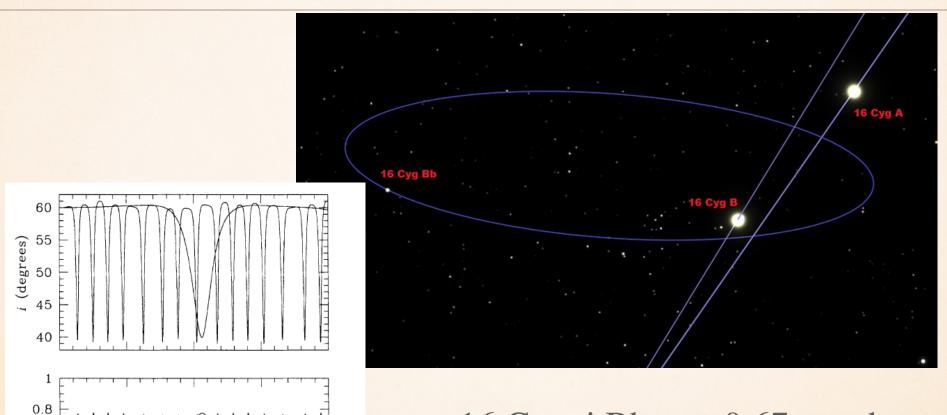
LK produces ~ 20% of the observed HJs

FORMATION OF HOT JUPITERS—OBSERVATIONAL EVIDENCES



16 Cygni Bb: e = 0.67

Cochran et al. 1996



0.6

0.2

0 [

5000

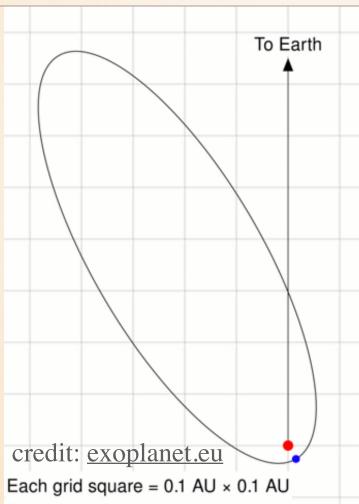
 10^{4}

t (binary periods)

1.5×104

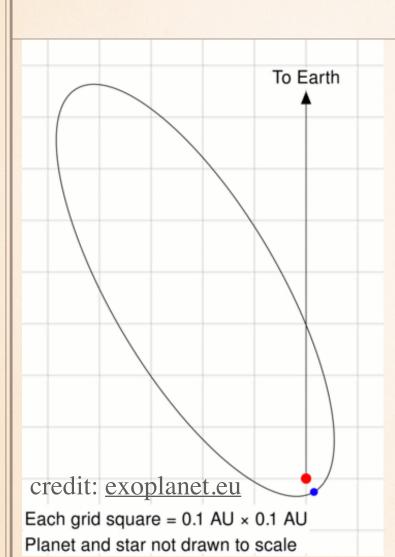
16 Cygni Bb: e = 0.67, can be produced by Lidov-Kozai mechanism

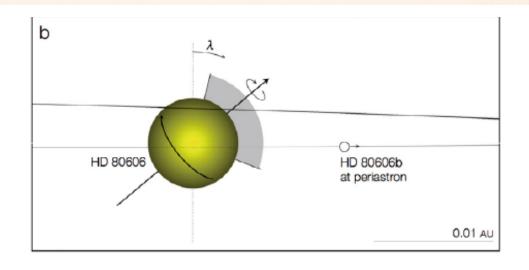
Holman et al. 1997



Planet and star not drawn to scale

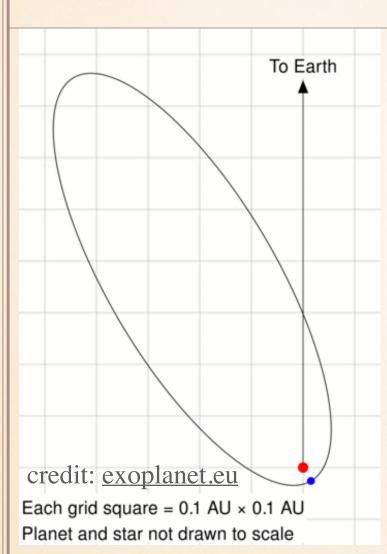
Naef et al. 2001

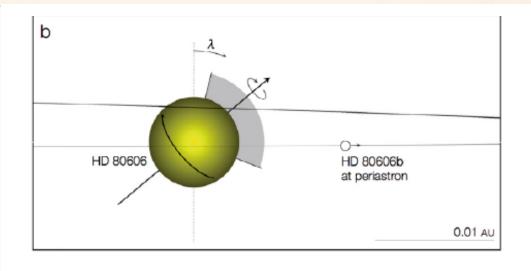




Pont et al. 2009

Naef et al. 2001





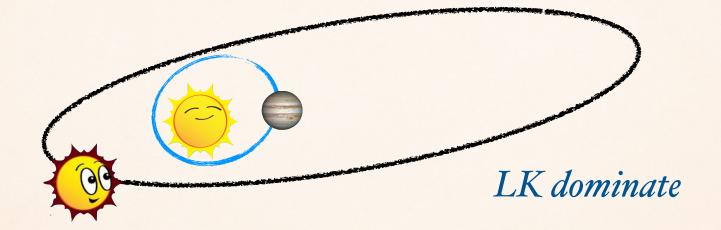
Pont et al. 2009

HD80606b: e = 0.93, can be produced by Lidov-Kozai mechanism Wu & Murray 2003

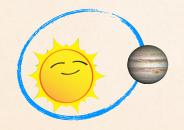
Naef et al. 2001

FRIENDS OF HOT JUPITERS

Existence an outer companion?



or



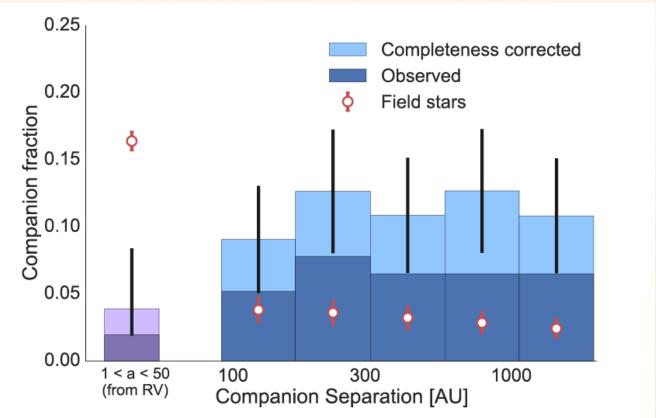
LK not dominate

Knutson et al. 2014

FRIENDS OF HOT JUPITERS

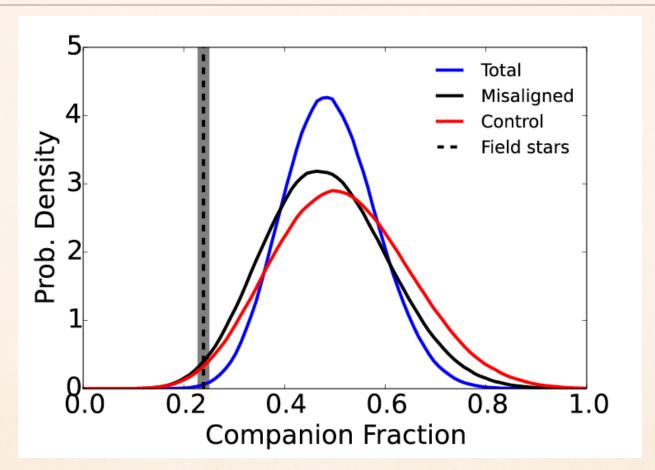
47%±7% of hot Jupiter have stellar companions with a b.t. 50-2000 AU based on 77 transiting hot Jupiters

Ngo et al. 2016



< 16%±5% systems formed via Lidov-Kozai oscillations

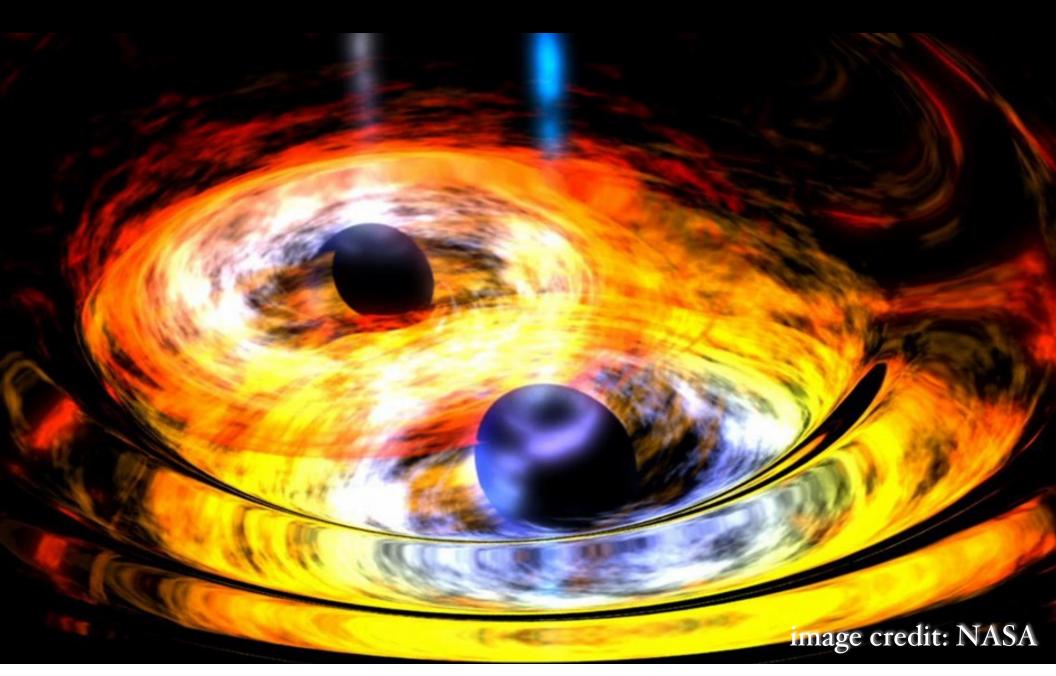
FRIENDS OF HOT JUPITERS



No correlation between misaligned/eccentric hot Jupiter systems and the incidence of stellar companions based on 27 misaligned/eccentric HJs

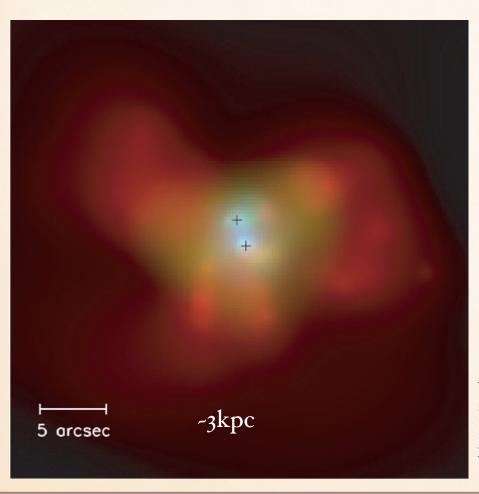
Ngo et al. 2015

EXAMPLES --- 2. EFFECTS ON STARS SURROUNDING SMBHB



EXAMPLES --- 2. EFFECTS ON STARS SURROUNDING SMBHB

SMBHBs originate from mergers between galaxies.



• SMBHBs with mostly -kpc separation have been observed with direct imagine.

(e.g., Woo et al. 2014; Komossa et al. 2013, Fabbiano et al. 2011, Green et al. 2010, Civano et al. 2010, Rodriguez et al. 2006, Komossa et al. 2003, Hutchings & Neff 1989)

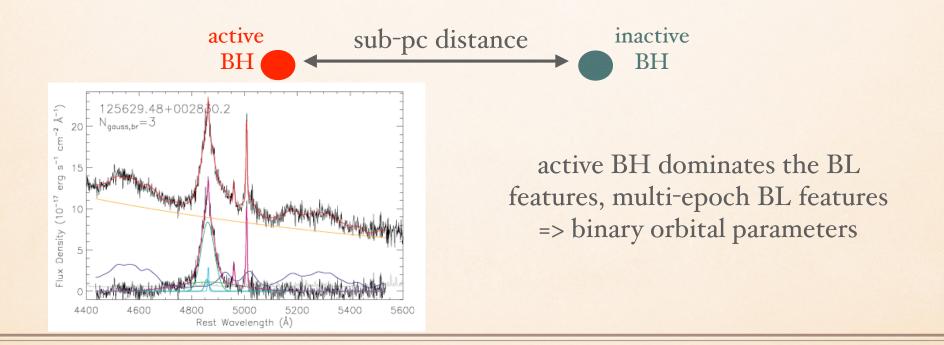
Multicolor image of NGC 6240. Red p soft (0.5–1.5 keV), green p medium (1.5–5 keV), and blue p hard (5–8 keV) X-ray band. (Komossa et al. 2003)

STARS SURROUNDING SMBHB

 At -ipc separation it is more difficult to identify SMBHBs. SMBHBs can be observed with photometric and spectral features.

(e.g., Shen et al. 2013, Boroson & Lauer 2009, Valtonen et al. 2008, Loeb 2007)

Example of multi-epoch spectroscopy (Shen et al. 2013):



STARS SURROUNDING SMBHB

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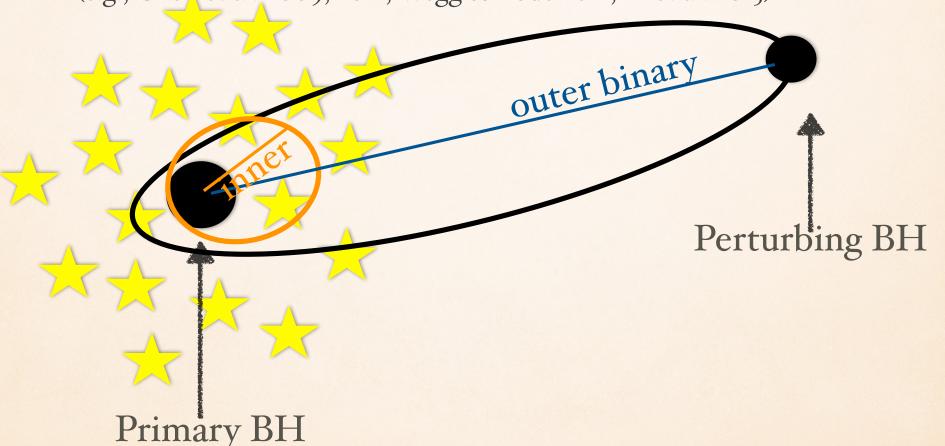
• Identify SMBHB at -1 pc separation by stellar features due to interactions with SMBHB.

(e.g., Chen et al. 2009, 2011, Wegg & Bode 2011, Li et al. 2015)

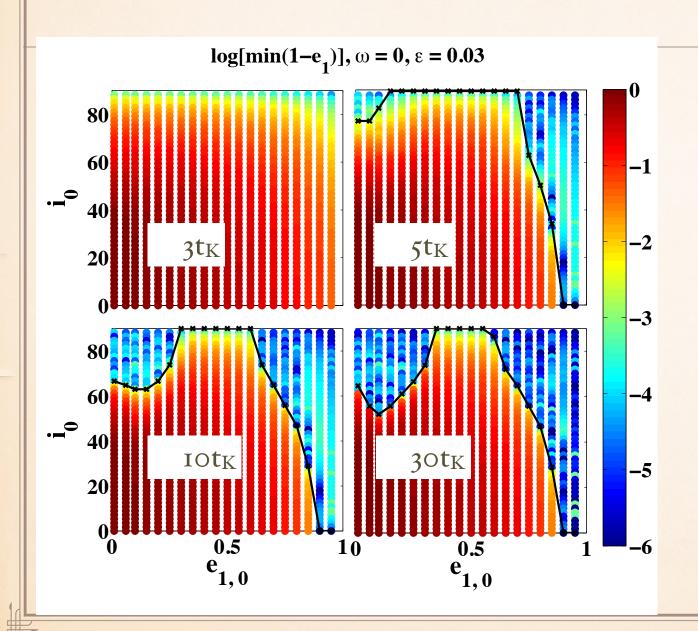
PERTURBATIONS ON STARS SURROUNDING SMBHB

• Identify SMBHB at -1 pc separation by stellar features due to interactions with SMBHB.

(e.g., Chen et al. 2009, 2011, Wegg & Bode 2011, Li et al. 2015)



ENHANCEMENT OF TIDAL DISRUPTION RATES



 $e_{I, \text{max}}$ determines the closest distance:

$$r_p \propto (I-e_I)$$

$$t_K = \frac{8}{3} P_{in} \frac{m_1}{m_2} \left(\frac{a_2}{a_1}\right)^3 (1 - e_2^2)^{3/2}$$

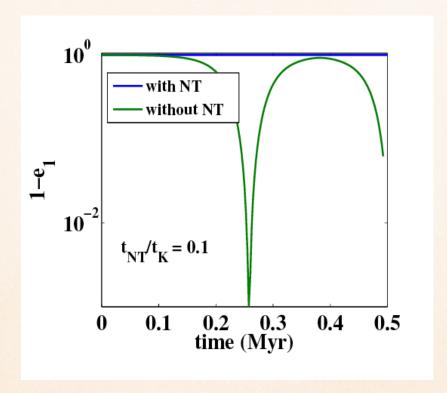
e_{max} reaches 1-10⁻⁶ over -30t_K (-Myrs)

Starting at *a*-10⁶R_t, it's still possible to be disrupted in -30t_K!

Li et al. 2014a

SUPPRESSION OF ELK

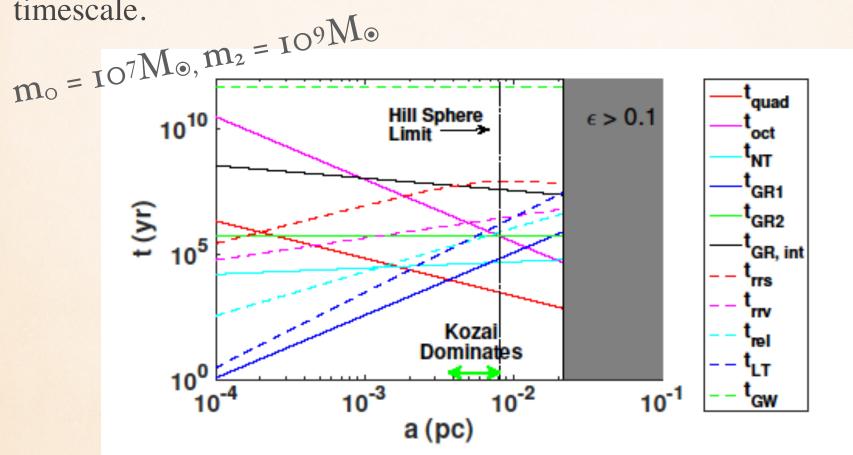
 Eccentricity excitation suppressed when precession timescale < Kozai timescale.



 $m_0 = 10^7 M_{\odot}$, $m_2 = 10^9 M_{\odot}$, $e_1 = 2/3$, $a_2 = 0.3$ pc, $m_1 = 1 M_{\odot}$, $e_2 = 0.7$. (Li et al. 2015)

SUPPRESSION OF ELK

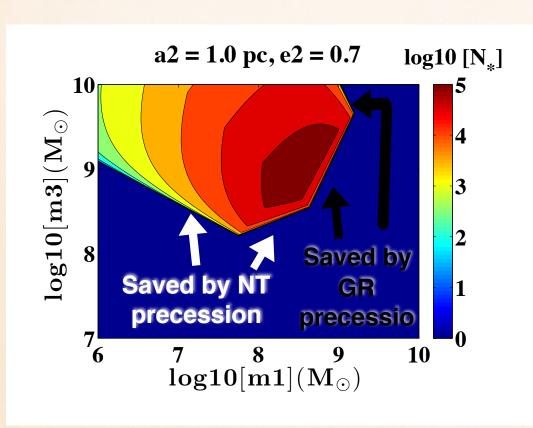
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 $e_1 = 2/3$, $a_2 = 0.3$ pc, $m_1 = 1M_{\odot}$, $e_2 = 0.7$.

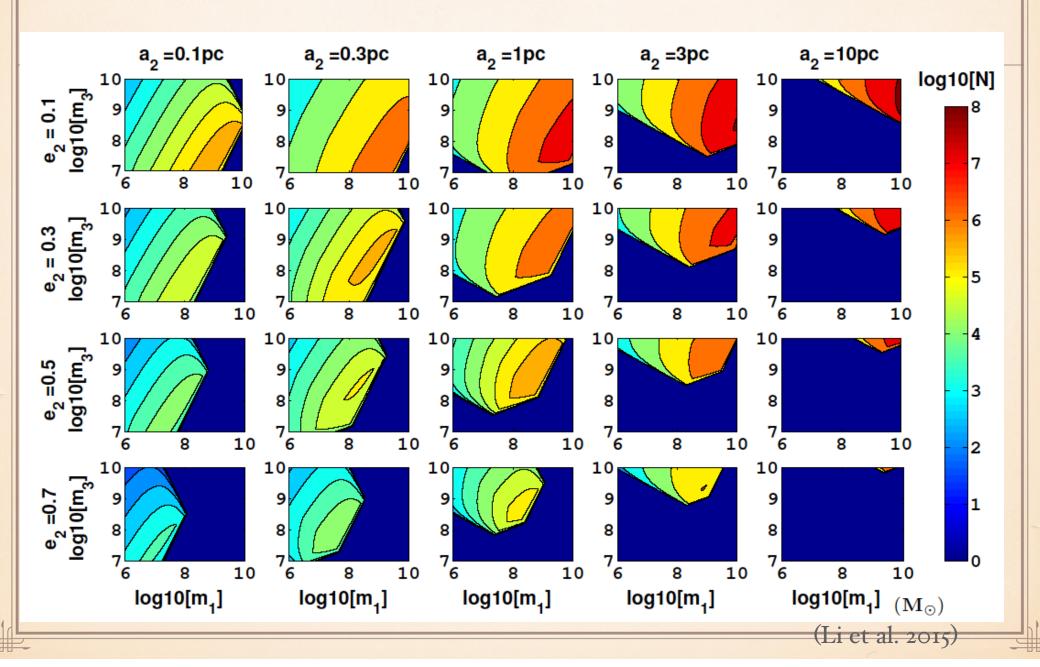
EXAMPLES --- 2. EFFECTS ON STARS SURROUNDING SMBHB

Eccentricity excitation suppressed when precession timescale
 Kozai timescale.



Kozai affects
 more stars when
 perturbing more
 massive SMBH.

SUPPRESSION OF ELK

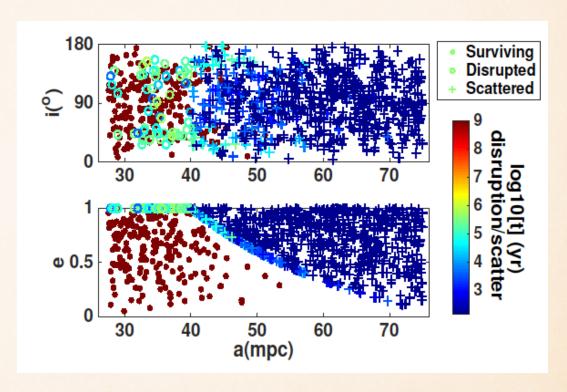


EXAMPLES --- 2. EFFECTS ON STARS SURROUNDING SMBHB

• 57/1000 disrupted; 726/1000 scattered.

=> Scattered stars may change the stellar density profile around the SMBH to the shape of a donut.

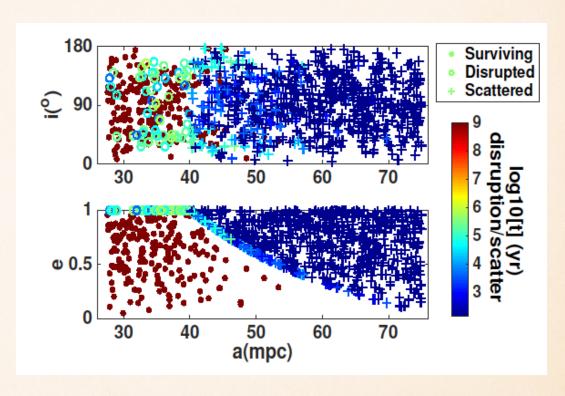




• Example: $m_1 = 10^7 \,\mathrm{M}_{\odot}$, $m_2 = 10^8 \,\mathrm{M}_{\odot}$, $a_2 = 0.5 \,\mathrm{pc}$, $e_2 = 0.5$, Run time: 1Gyr.

EXAMPLES --- 2. EFFECTS ON STARS SURROUNDING SMBHB

- 57/1000 disrupted; 726/1000 scattered.
- => Scattered stars may change stellar density profile around the SMBH.
- => Disruption rate can reach ~10⁻³/yr.



• Example: $m_1 = 10^7 \,\mathrm{M}_{\odot}$, $m_2 = 10^8 \,\mathrm{M}_{\odot}$, $a_2 = 0.5 \,\mathrm{pc}$, $e_2 = 0.5$, Run time: 1Gyr.

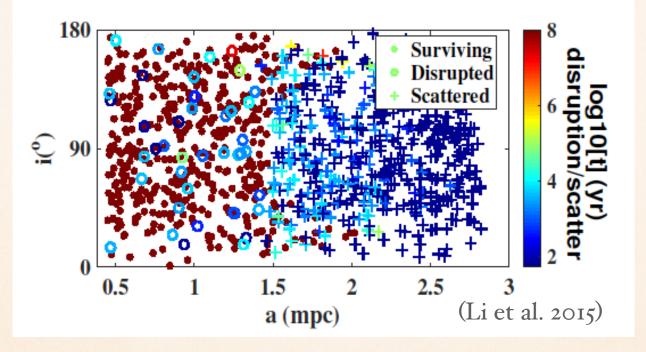
• Example: $m_1 = 10^4 \,\mathrm{M}_{\,\odot}$, $m_2 = 4 \times 10^6 \,\mathrm{M}_{\,\odot}$, $a_2 = 0.1 \,\mathrm{pc}$, $e_2 = 0.7$ (Run time: 100 Myr)





• Example: $m_1 = 10^4 \,\mathrm{M}_{\odot}$, $m_2 = 4 \times 10^6 \,\mathrm{M}_{\odot}$, $a_2 = 0.1 \,\mathrm{pc}$, $e_2 = 0.7$ (Run time: 100)

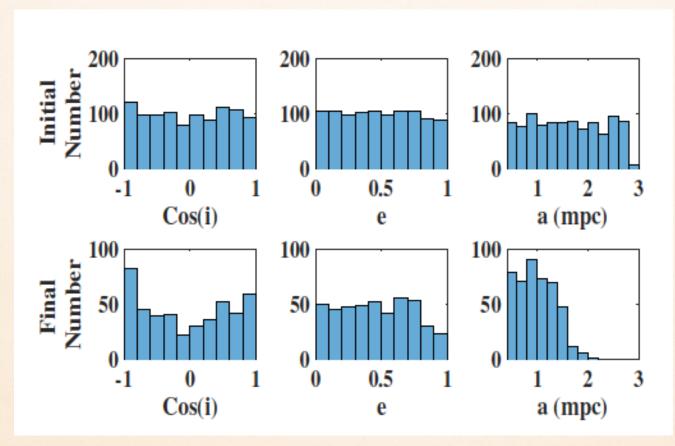
Myr)



• 40/1000 disrupted; 500/1000 scattered.

- => ~50% stars survived.
- => Disruption rate can reach ~10⁻⁴/yr.

• Example: $m_1 = 10^4 \,\mathrm{M}_{\,\odot}$, $m_2 = 4 \times 10^6 \,\mathrm{M}_{\,\odot}$, $a_2 = 0.1 \,\mathrm{pc}$, $e_2 = 0.7$, $\alpha = 1.75$ (Runtime: 100Myr)



CONCLUSION

- Perturbation of the outer object can produce flips of the inner orbit and excite inner orbit eccentricity
- Under tidal dissipation, the perturbation of a farther companion can produce misaligned hot Jupiters
- Perturbation of a SMBH may enhance the tidal disruption rate of stars.





Systematic Study of the Parameter Space

• Identify the resonances and the chaotic region.

• Characterize the parameter space that give rise to the interesting behaviors — eccentricity excitation and orbital flips.

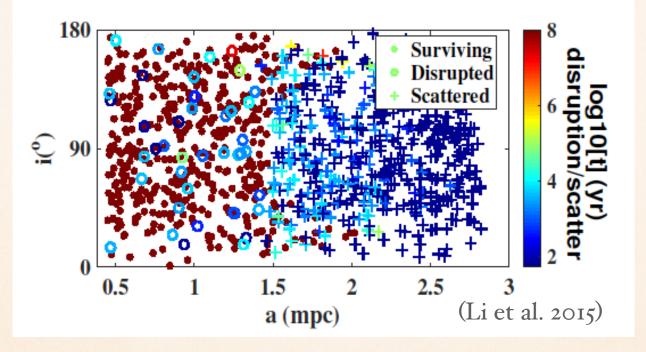
• Example: $m_1 = 10^4 \,\mathrm{M}_{\,\odot}$, $m_2 = 4 \times 10^6 \,\mathrm{M}_{\,\odot}$, $a_2 = 0.1 \,\mathrm{pc}$, $e_2 = 0.7$ (Run time: 100 Myr)





• Example: $m_1 = 10^4 \,\mathrm{M}_{\odot}$, $m_2 = 4 \times 10^6 \,\mathrm{M}_{\odot}$, $a_2 = 0.1 \,\mathrm{pc}$, $e_2 = 0.7$ (Run time: 100)

Myr)

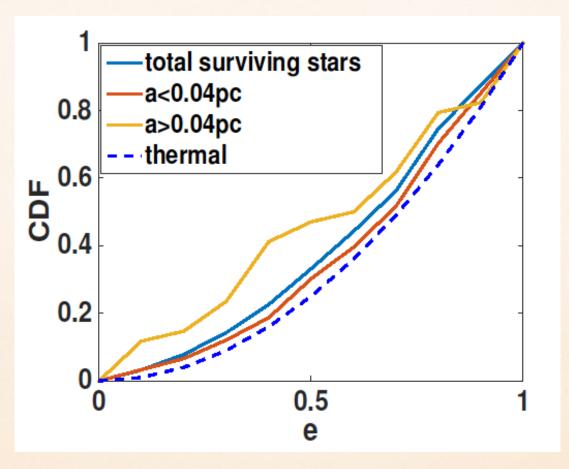


• 40/1000 disrupted; 500/1000 scattered.

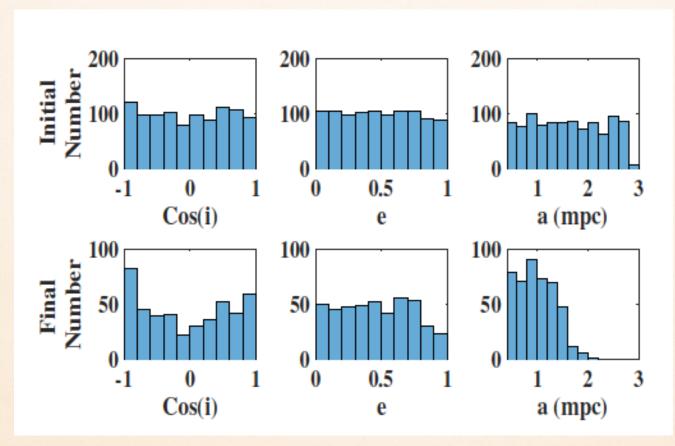
- => ~50% stars survived.
- => Disruption rate can reach ~10⁻⁴/yr.

EFFECTS OF EKM ON STARS SURROUNDING BBH

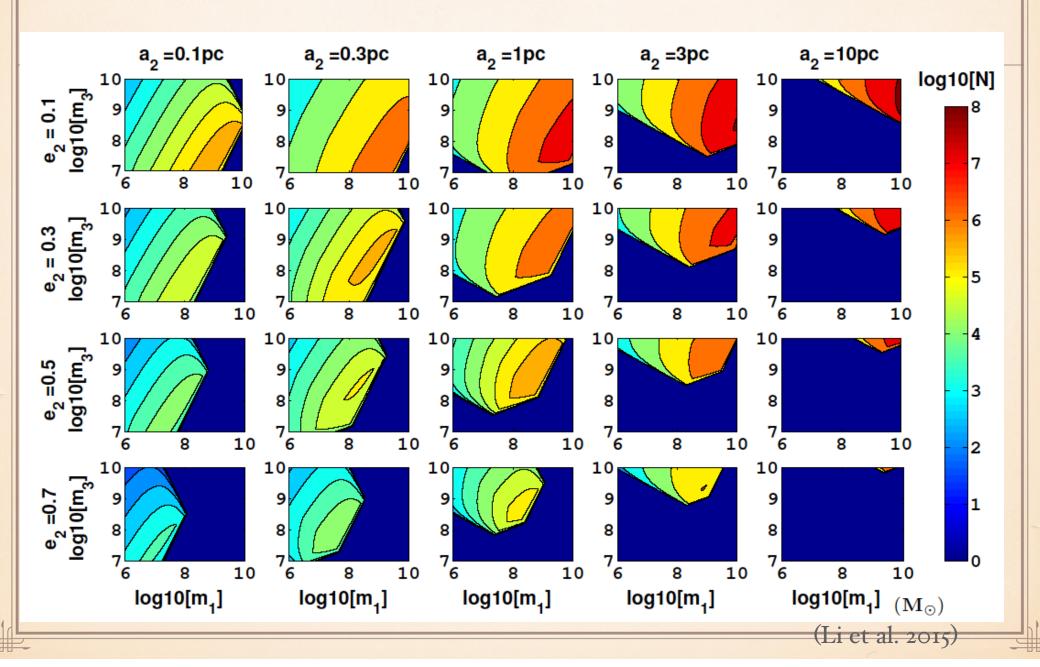
• Example: $m_1 = 10^7 \,\mathrm{M}_{\odot}$, $m_2 = 10^8 \,\mathrm{M}_{\odot}$, $a_2 = 0.5 \,\mathrm{pc}$, $e_2 = 0.5$, $\alpha = 1.75$. Run time: 1Gyr.



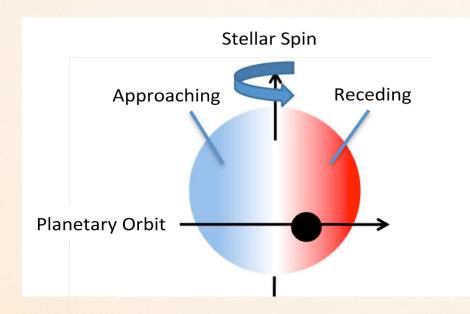
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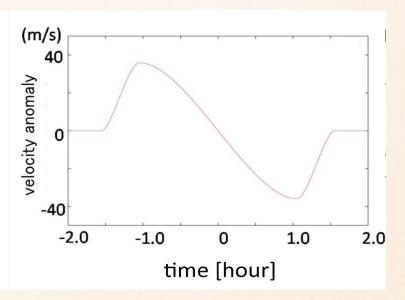


SUPPRESSION OF ELK

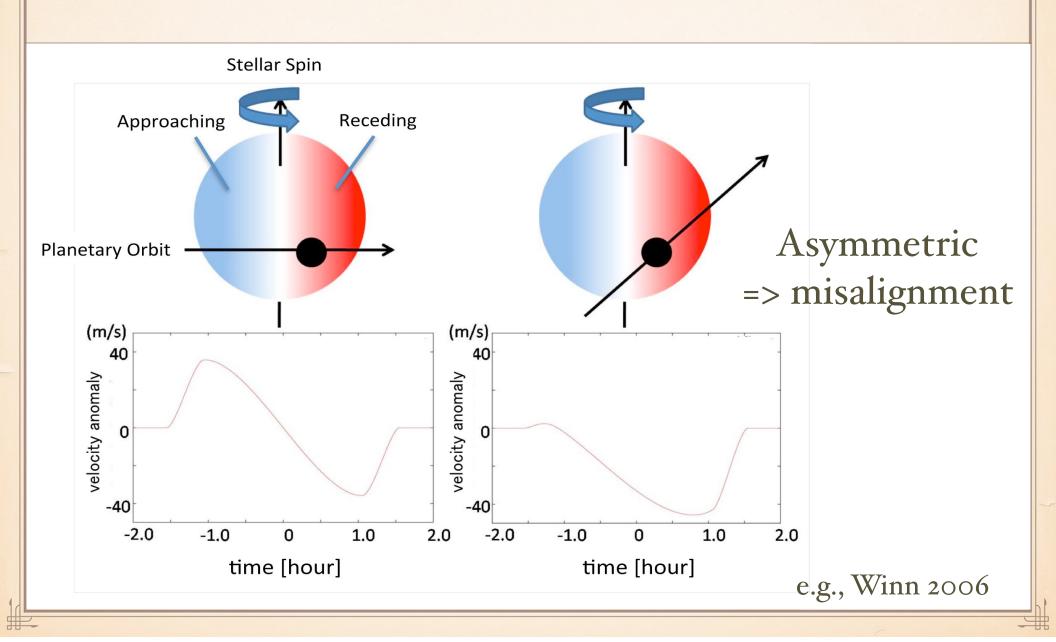


ROSSITER-MCLAUGHLIN METHOD (SPIN-ORBIT MISALIGNMENT)



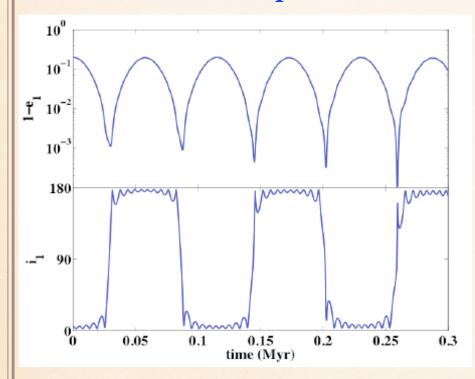


ROSSITER-MCLAUGHLIN METHOD (SPIN-ORBIT MISALIGNMENT)

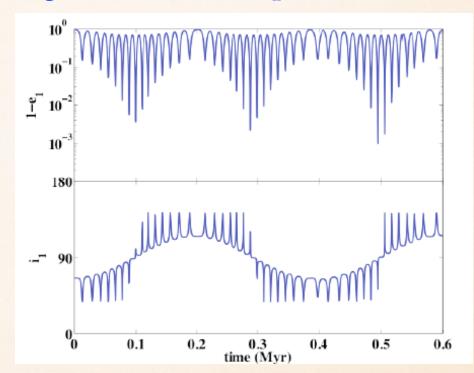


DIFFERENCES BETWEEN HIGH/LOW I FLIP

Low inclination flip



High inclination flip



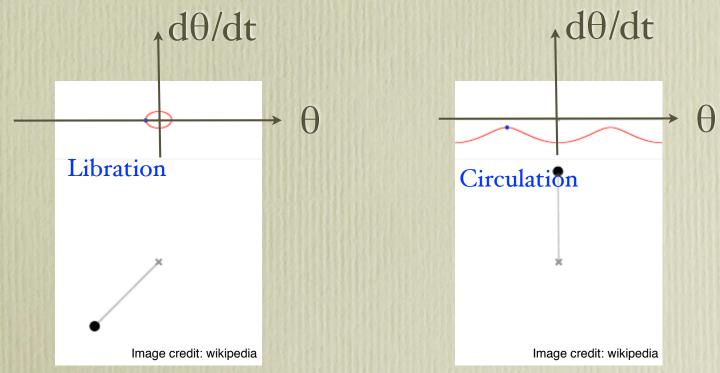
Low inclination flips:

- e₁ † monotonically, inclination stays low before flip.
- Flip occurs faster.

(Li et al. 2014a)

Resonances and Chaotic Regions

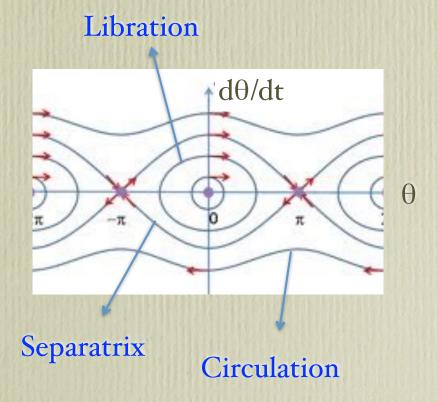
- The Hamiltonian H_{res} takes form of a pendulum.
- Two dynamical regions: libration region and circulation region.



Resonances and Chaotic Regions

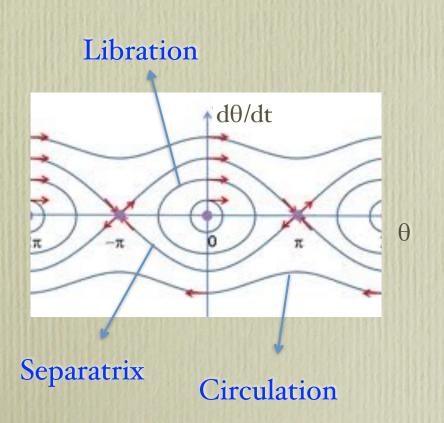
- The Hamiltonian H_{res} takes form of a pendulum.
- Two dynamical regions: libration region and circulation region, separated by separatrix.

Phase Diagram:

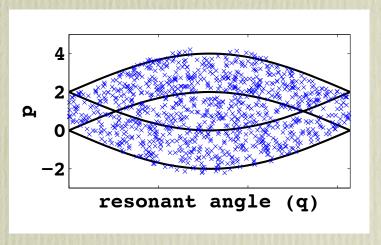


Resonances and Chaotic Regions

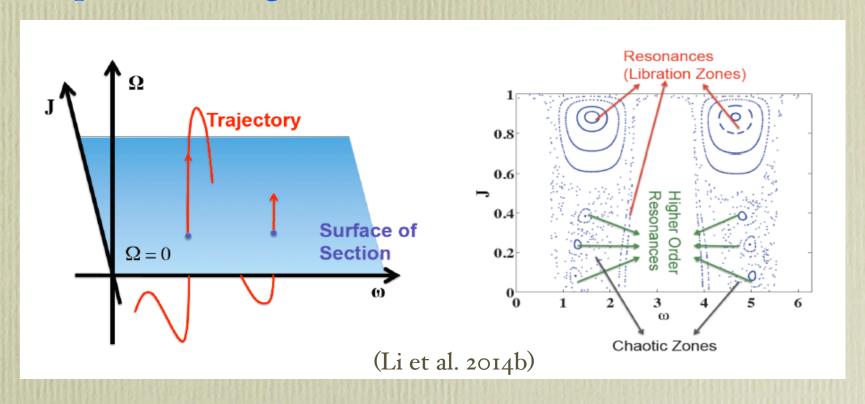
- The Hamiltonian H_{res} takes form of a pendulum.
- Two dynamical regions: libration region and circulation region, separated by separatrix.



Overlap of resonances can cause chaos

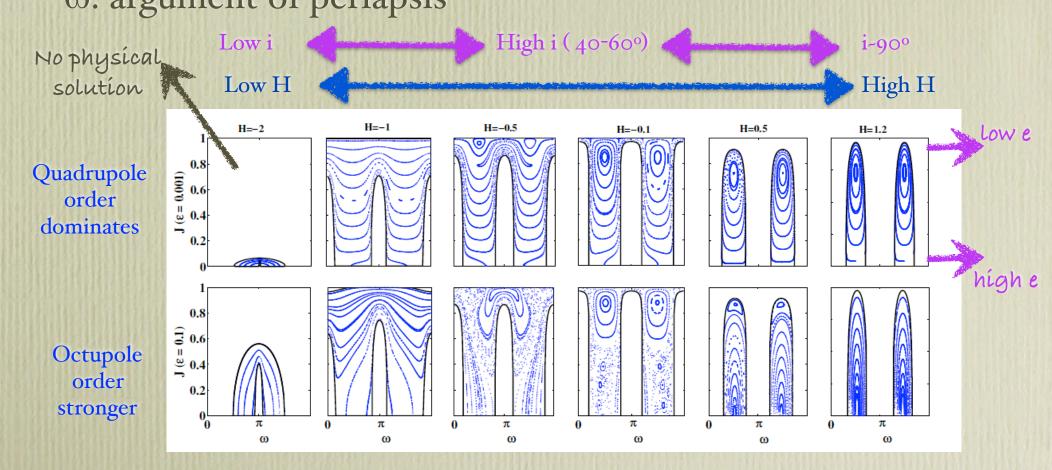


Example of a 2-degree freedom H (J, ω , Jz, Ω)

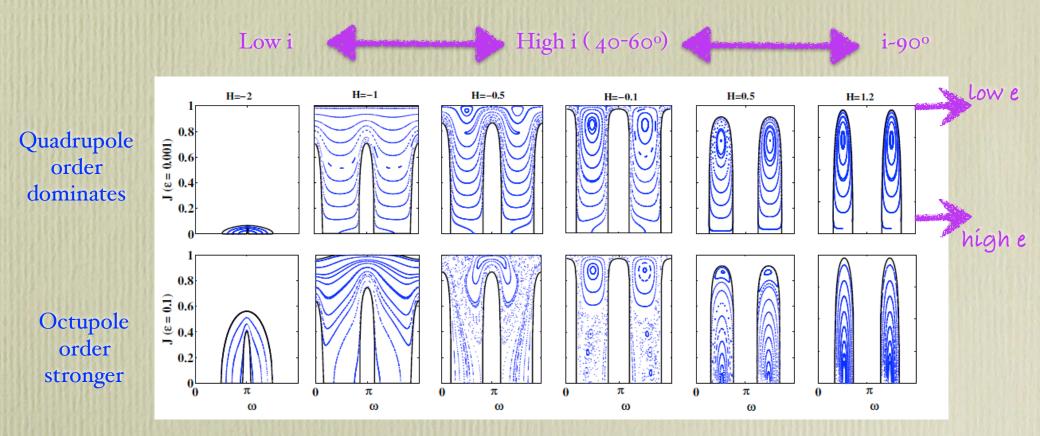


- Resonant zones: points fill 1-D lines. trajectories are quasi-periodic.
- Chaotic zones: points fill a higher dimension.

- Surface of section of hierarchical three-body problem in the test particle limit in the $J-\omega$ Plane.
- $J = \sqrt{1 e_1^2}$ (specific angular momentum); ω : argument of periapsis



Resonances exist for all surfaces:



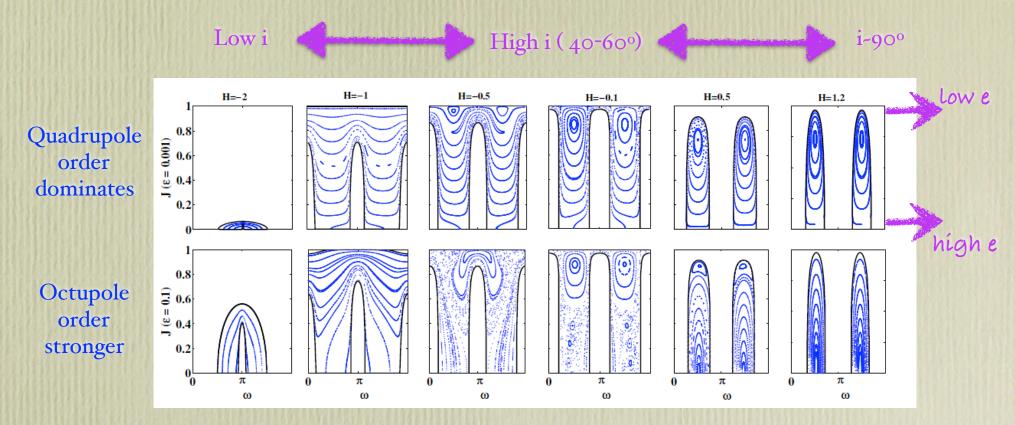
Quadrupole resonances:

centers at low e_1 , $\omega = \pi/2$ and $3\pi/2$ (e.g. Kozai 1962)

Octupole resonances:

centers at high e_1 , $\omega = \pi$ or $\pi/2$ and $3\pi/2$

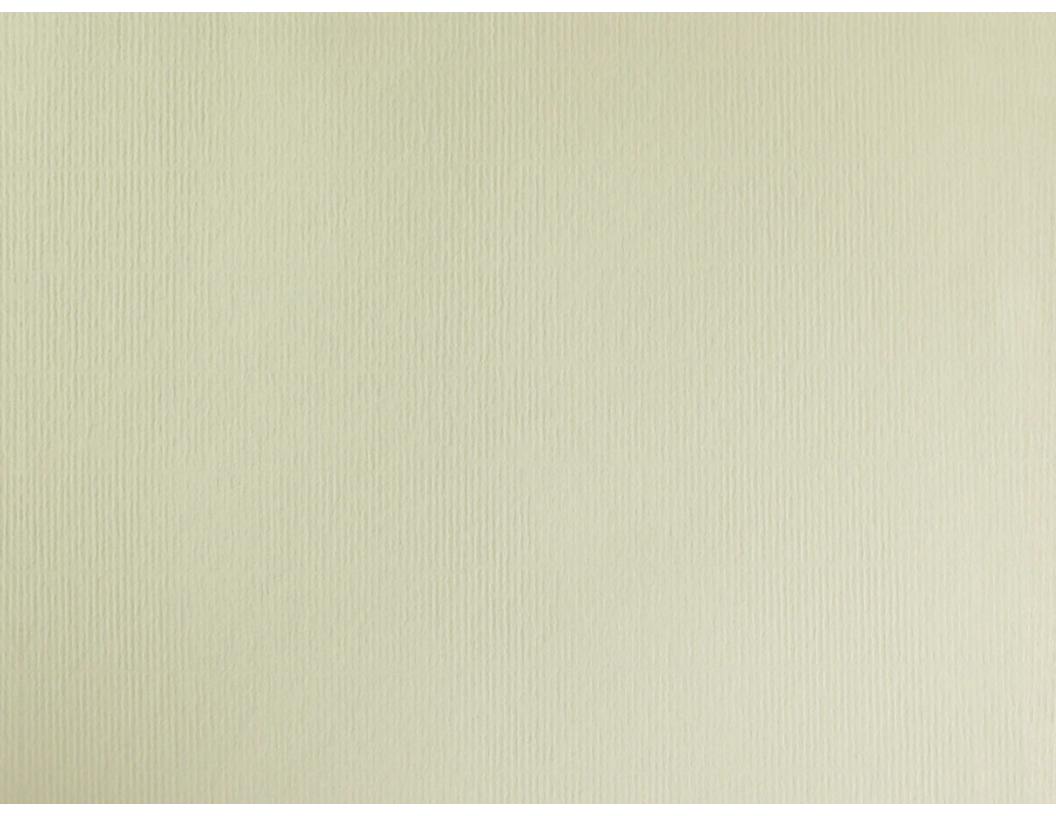
Li et al. 2014b



- e_i excitation ($J \rightarrow o$) are caused by octupole resonances.
- · Near coplanar flip due to octupole resonances alone.
- High inclination flip due to both quadrupole and octupole order resonances.

Summary

- Hierarchical Three Body Dynamics:
 - Starting with near coplanar configuration, the inner orbit of a hierarchical 3-body system can flip by -180° , and $e_{\tau} \rightarrow 1$.
 - This mechanism is regular, and the flip criterion and timescale can be expressed analytically.
 - This mechanism can produce counter orbiting hot exoplanets, and can enhance collision/tidal disruption rate.
- Underlying resonances:
 - Flips and e₁ excitations are caused by octupole resonances.
 - High inclination flips are chaotic, with Lyapunov timescale
 6t_K.



Summary

• Coplanar flip:

- Starting with near coplanar configuration, the inner orbit of a hierarchical 3-body system can flip by -180° , and $e_{\tau} \rightarrow 1$.
- This mechanism is regular, and the flip criterion and timescale can be expressed analytically.
- This mechanism can produce counter orbiting hot exoplanets, and can enhance collision/tidal disruption rate.

• Characterization of parameter space:

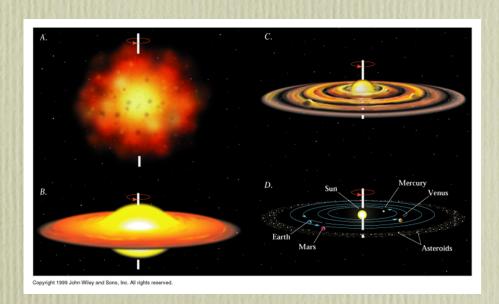
- Near coplanar flip and e₁ excitations are caused by octupole resonances.
- High inclination flips are chaotic, with Lyapunov timescale
 6t_K.

Potential Applications

- Captured stars in BBH systems may affect stellar distribution around the BHs (e.g., Ann-Marie Madigan, Smadar Naoz, Ryan O'Leary).
- Tidal disruption and collision events for planetary systems (e.g., Eugene Chiang, Bekki Dawson, Smadar Naoz).
- Production of supernova (e.g., Rodrigo Fernandez, Boaz Katz, Todd Thompson).
- Other aspects:
 - Involving more bodies (e.g., Smadar Naoz, Todd Thompson).
 - Obliquity variation of planets.

COHJ Contradict with popular Planets' Formation Theory

• Formation Theory:



- Planet systems form from cloud contraction.
- Spin of the star ends up aligned with the orbit of the planets

Analytical Overview --- Test Particle Limit

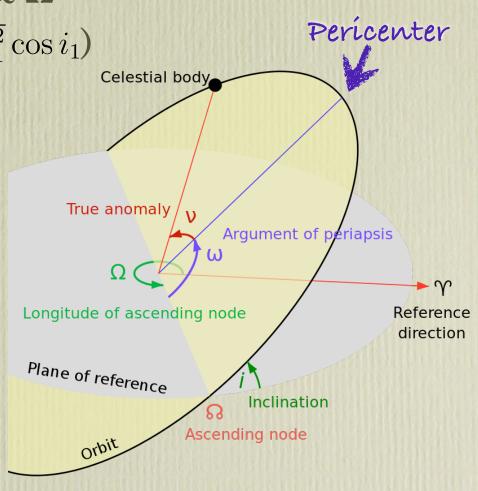
• Hamiltonian has two degrees of freedom:

2 conjugate pairs: J & ω , Jz & Ω

$$(J = \sqrt{1 - e_1^2}, Jz = \sqrt{1 - e_1^2} \cos i_1)$$

ω: orientation in orbital plane.

 Ω : orientation in reference plane.



Analytical Overview

- Hamiltonian (Harrington 1968, 1969; Ford et al., 2000):
 - In the octupole order: $H = -F_{quad} \varepsilon F_{oct}$, $\varepsilon = (a_1/a_2)e_2/(1-e_2^2)$

$$F_{quad} = -(e_1^2/2) + \theta^2 + 3/2e_1^2\theta^2 + 5/2e_1^2(1 - \theta^2)\cos(2\omega_1),$$

$$F_{oct} = \frac{5}{16}(e_1 + (3e_1^3)/4) \times ((1 - 11\theta - 5\theta^2 + 15\theta^3)\cos(\omega_1 - \Omega_1) + (1 + 11\theta - 5\theta^2 - 15\theta^3)\cos(\omega_1 + \Omega_1)) - \frac{175}{64}e_1^3((1 - \theta - \theta^2 + \theta^3)\cos(3\omega_1 - \Omega_1) + (1 + \theta - \theta^2 - \theta^3)\cos(3\omega_1 + \Omega_1)),$$

• Independent of Ω_{I} , J_{z} const.

Depend on both ω₁ and Ω₁
 ⇒ both J and J_z are not const.

$$t_K = \frac{8}{3} P_{in} \frac{m_1}{m_2} \left(\frac{a_2}{a_1}\right)^3 (1 - e_2^2)^{3/2}$$

Analytical Derivation for Flip Criterion and Timescale

- Hamiltonian (at O(i)):
 - Evolution of e₁ only due to octupole terms: => e₁ does not oscillate before flip.
 - Depend on only J_r and $\varpi_r = \omega_r + \Omega_r$ => System is integrable. => $e_r(t)$ can be solved.
 - Flip at e_{I, max} I => The flip timescale can be derived.
 - Flip when $\varpi_1 = 180^{\circ}$ => The flip criterion can be derived.

$$\varepsilon > \frac{8}{5} \frac{1 - e_1^2}{7 - e_1 (4 + 3e_1^2) \cos(\omega_1 + \Omega_1)}$$

Analytical Overview

• Hamiltonian has two degrees of freedom:

$$(J = \sqrt{1 - e_1^2}, Jz = \sqrt{1 - e_1^2} \cos i_1, \omega, \Omega)$$

2 conjugate pairs: J & ω , Jz & Ω

• Hamiltonian (Harrington 1968, 1969; Ford et al. 2000):

In the octupole order:

Interaction Energy (H) of two orbital wires:

$$H = F_{quad}(J, Jz, \omega) + \epsilon F_{oct}(J, Jz, \omega, \Omega)$$

Quadrupole order: Independent of Ω => Jz constant

←: hierarchical parameter:

$$\epsilon = \frac{a_1}{a_2} \frac{e_2}{1 - e_2^2}$$

Octupole order: Depend on both $\Omega \& \omega \Rightarrow J$ and Jz not constant

Analytical Der

put equation in hidden slides

Flip Criterion le

ar

- Hamiltonian (at O(i)) depend on only e_1 and $\varpi_1 = \omega_1 + \Omega_1$:
- Evolution of e₁ only due to octupole terms:

$$\dot{e}_1 = \frac{5}{8} J_1 (3J_1^2 - 7) \varepsilon \sin(\varpi_1) \qquad \dot{\varpi}_1 = J_1 \left(2 + \frac{5(9J_1^2 - 13)\varepsilon \cos(\varpi_1)}{\sqrt{1 - J_1^2}} \right)$$

• e₁(t) can be solved =>

The flip criterion and the flip timescale can be derived:

$$\varepsilon > \frac{8}{5} \frac{1 - e_1^2}{7 - e_1 (4 + 3e_1^2) \cos(\omega_1 + \Omega_1)}$$

FLIP CRITERION

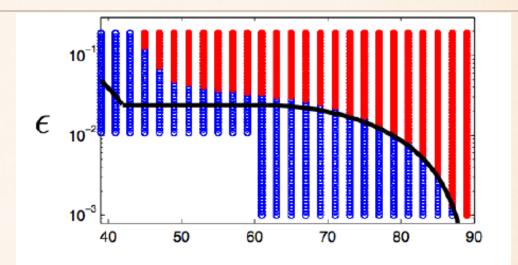
Averaging the quadrupole oscillations in limit $j_z \sim 0$, Katz et al. 2011 obtain the constant:

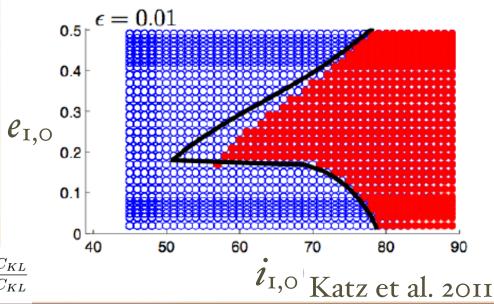
$$f(C_{KL}) + \epsilon \frac{\cos i_{\text{tot}} \sin \Omega_1 \sin \omega_1 - \cos \omega_1 \cos \Omega_1}{\sqrt{1 - \sin^2 i_{\text{tot}} \sin^2 \omega_1}}$$

 \bigcirc Requiring $j_z = 0$, during the flip:

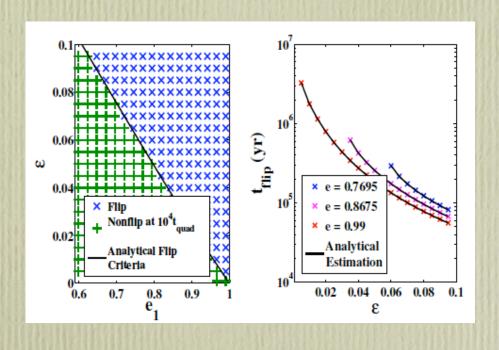
$$\epsilon_c = \frac{1}{2} f\left(\frac{1}{2}\cos^2 i_{\text{tot},0}\right)$$

$$f(C_{KL}) = \frac{32\sqrt{3}}{\pi} \int_{x_{min}}^{1} \frac{K(x) - 2E(x)}{(41x - 21)\sqrt{2x + 3}} dx$$
 and $x_{min} = \frac{3 - 3C_{KL}}{3 + 2C_{KL}}$





Analytical Results v.s. Numerical Results

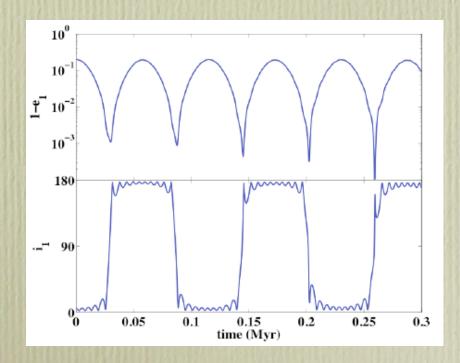


Why do analytical results with low inclination approximation work?

IC:
$$m_{I} = IM_{\odot}$$
, $m_{2} = 0.IM_{\odot}$, $a_{I} = IAU$, $a_{2} = 45.7AU$, $\omega_{I} = 0^{\circ}$, $\Omega_{I} = 180^{\circ}$, $i_{I} = 5^{\circ}$.

Analytical Results v.s. Numerical Results

Why do analytical results with low inclination approximation work?



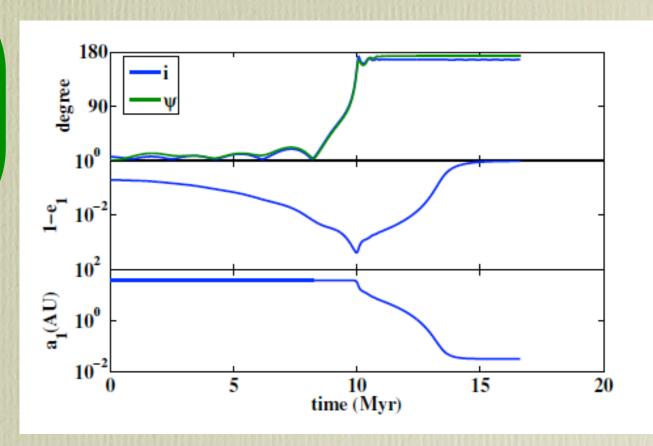
Small inclination assumption holds for most of the evolution.

IC:
$$m_1 = 1 M_{\odot}$$
, $m_3 = 1M_3$, $m_2 = 0.3 M_{\odot}$, $\omega_1 = 0^{\circ}$, $\Omega_1 = 180^{\circ}$, $e_2 = 0.6$, $a_1 = 4 AU$, $a_2 = 50 AU$, $e_1 = 0.8$, $i = 5^{\circ}$

Examples --- 1. Produce Counter Orbiting Hot Jupiters (+ tide)

Question:
Does this
mechanism produce
a peak at ψ≈180°?

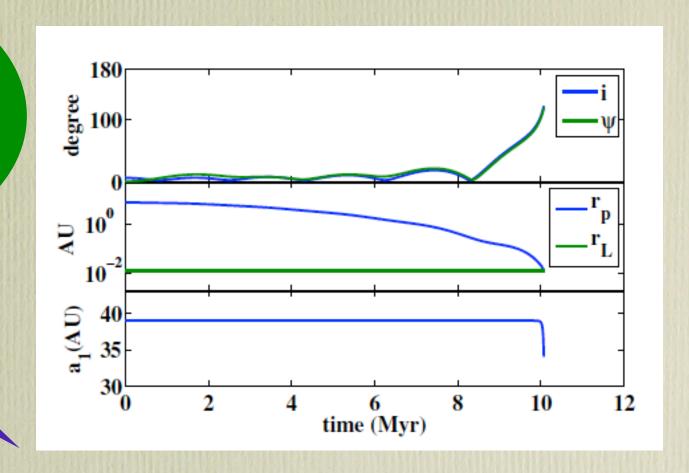
No.



Examples --- 1. Produce Counter Orbiting Hot Jupiters (+ tide)

Question:
Will planet be tidally disrupted?

Yes!



Applications --- 1. Produce Counter Orbiting Hot Jupiters (+ tide)

• Hot Jupiters:

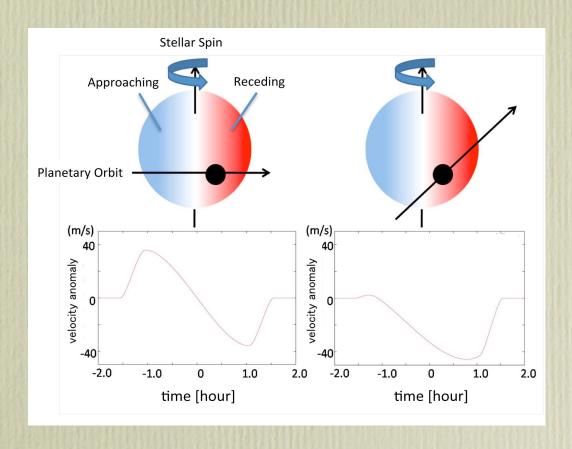
massive exoplanets (m ≥ m_J) with close-in orbits (period: 1-4 day).

• Counter Orbiting Hot Jupiters:

• Hot Jupiters that orbit in exactly the opposite direction to the spin of their host star.

• Disagree with the classical planet formation theory: the orbit aligns with the stellar spin.

Rossiter-McLaughlin Method



http://www.subarutelescope.org/

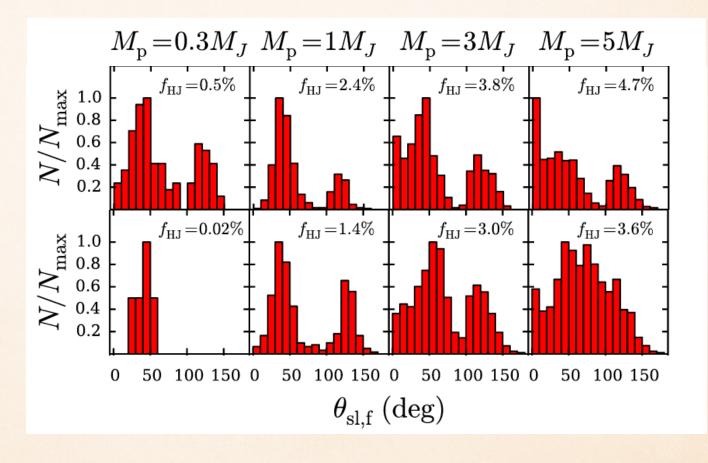
FORMATION OF MISALIGNED HOT JUPITERS (LK + STELLAR OBLATENESS + TIDE)

Anderson et al. 2016:

Mp < 3 M_J => bimodal

 $Mp \sim 5M_J$

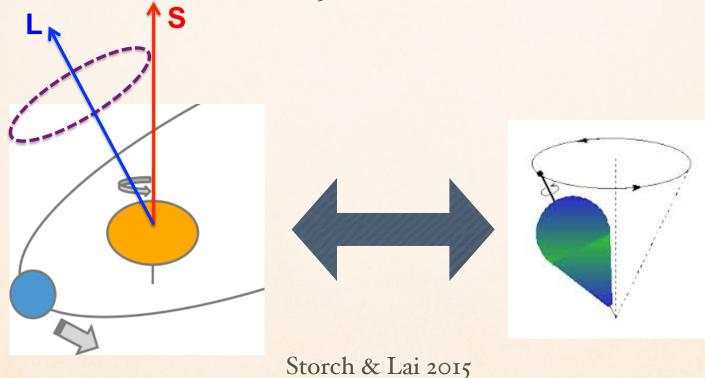
=> low
misalignment
(solar-type stars)
=> higher
misalignment
(more massive
stars)



Anderson et al. 2016

FORMATION OF MISALIGNED HOT JUPITERS (LK + STELLAR OBLATENESS + TIDE)

If the host star is spinning and oblate, gravity from the planet makes stellar spin precess around L, and can cause chaos under Lidov-Kozai oscillations (Storch & Lai 2015).



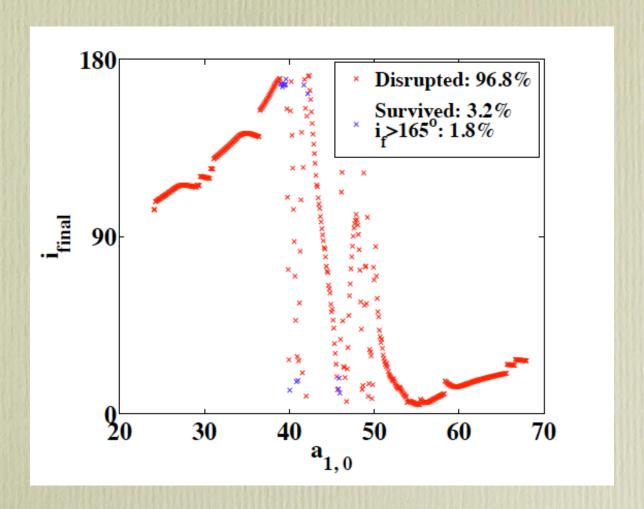
Chaos: precession period ~ Lidov-Kozai oscillation period

Take Home Message

• Eccentric Coplanar Kozai Mechanism can flip an eccentric coplanar inner orbit to produce counter orbiting exoplanets

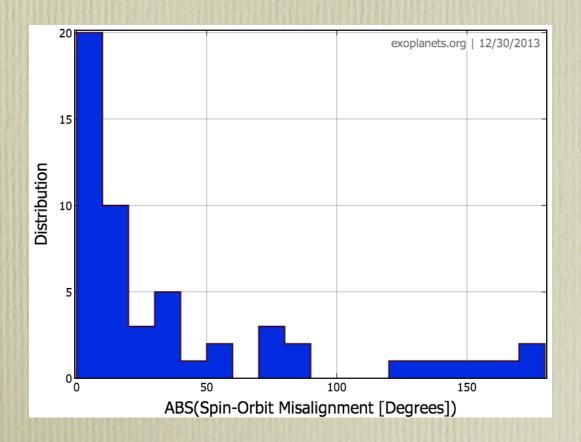


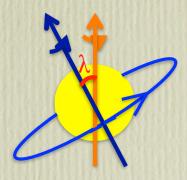
Eccentric inner orbit flips due to eccentric coplanar outer companion



Observational Links to Counter Orbiting Hot Jupiters

Distribution of sky projected spin-orbit angle
 (λ) of Hot Jupiters





There are retrograde hot jupiters (λ>90°)

It is possible to have counter orbiting planets.

Applications --- 2. Effects of EKM of Stars Surrounding BBH

• Tidal disruption rate is highly uncertain:

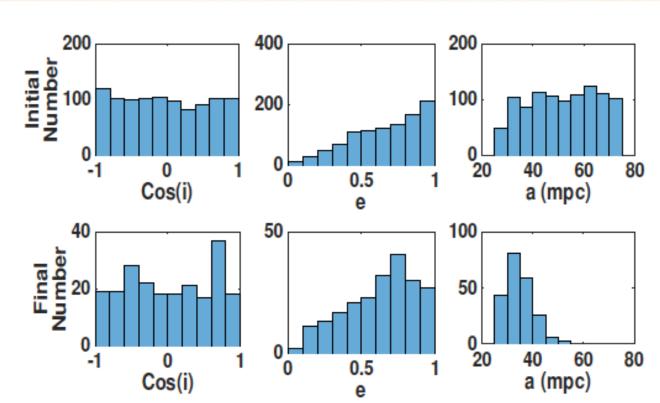
- It is observed to be 10-5--4/galaxy/yr from a very small sample by Gezari et al. 2008.
- It roughly agrees with theoretical estimates. (e.g. Wang & Merritt 2004)

• The disruption rate may be greatly enhanced:

- due to non-axial symmetric stellar potential. (Merritt & Poon 2004)
- due to SMBHB (Ivanov et al. 2005, Wegg & Bode 2011, Chen et al. 2011)
- due to recoiled SMBHB (Stone & Loeb 2011)

Examples --- 3. Effects of EKM of Stars Surrounding BBH

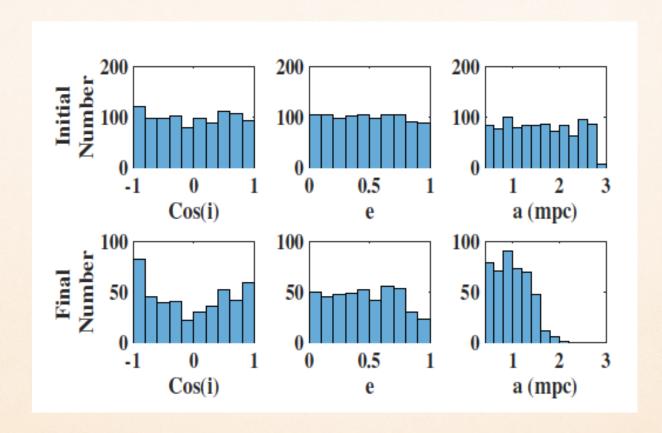
• Example: $m_1 = 10^7 \,\mathrm{M}_{\odot}$, $m_2 = 10^8 \,\mathrm{M}_{\odot}$, $a_2 = 0.5 \,\mathrm{pc}$, $e_2 = 0.5$, $\alpha = 1.75$ (stellar distribution), normalized by M- σ relation. Run time: 1Gyr.



(Li, et al. submitted 2015)

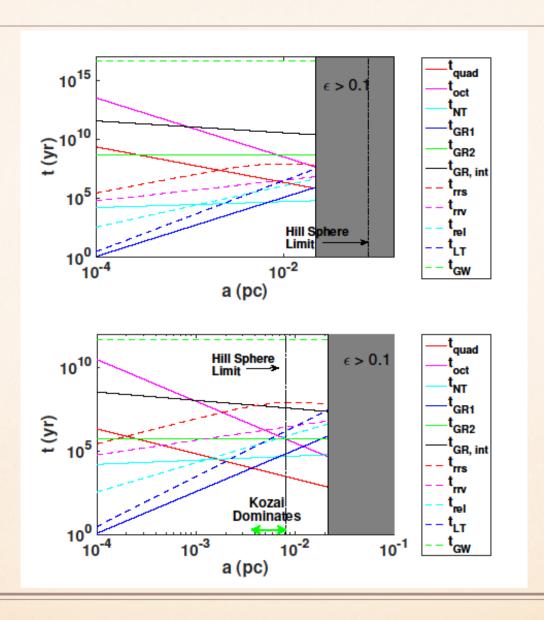
Examples --- 3. Effects of EKM of Stars Surrounding BBH

• Example: $m_1 = 10^4 \,\mathrm{M}_{\odot}$, $m_2 = 4 \times 10^6 \,\mathrm{M}_{\odot}$, $a_2 = 0.1 \,\mathrm{pc}$, $e_2 = 0.7$, $\alpha = 1.75$ (stellar distribution), normalized by M- σ relation. Run time: 100Myr.



(Li, et al. submitted 2015)

COMPARISON OF TIMESCALES

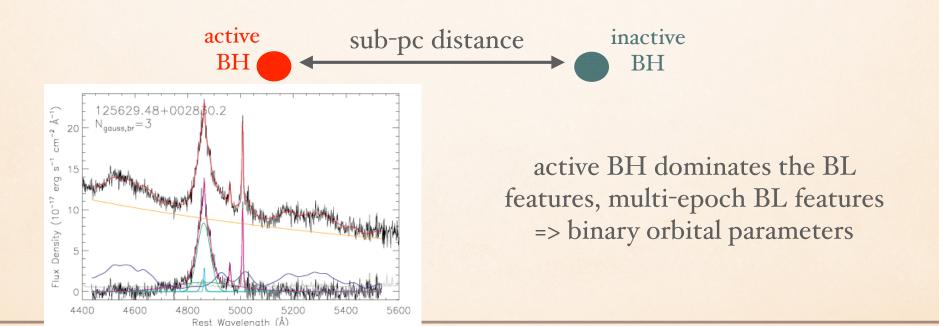


STARS SURROUNDING SMBHB

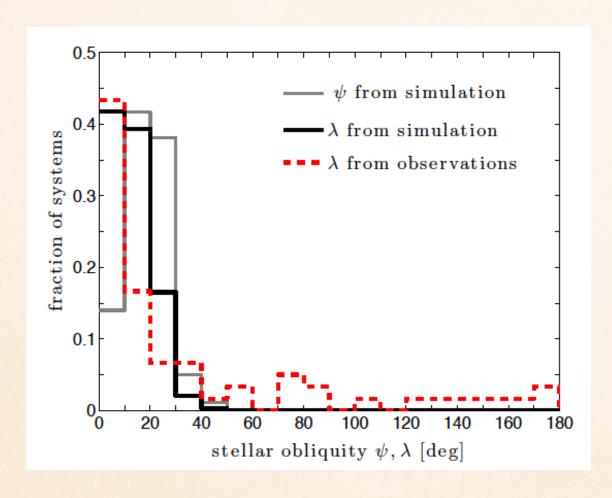
 At -ipc separation it is more difficult to identify SMBHBs. SMBHBs can be observed with spectral features.

(e.g., Shen et al. 2013, Boroson & Lauer 2009, Valtonen et al. 2008, Loeb 2007)

Example of multi-epoch spectroscopy (Shen et al. 2013):



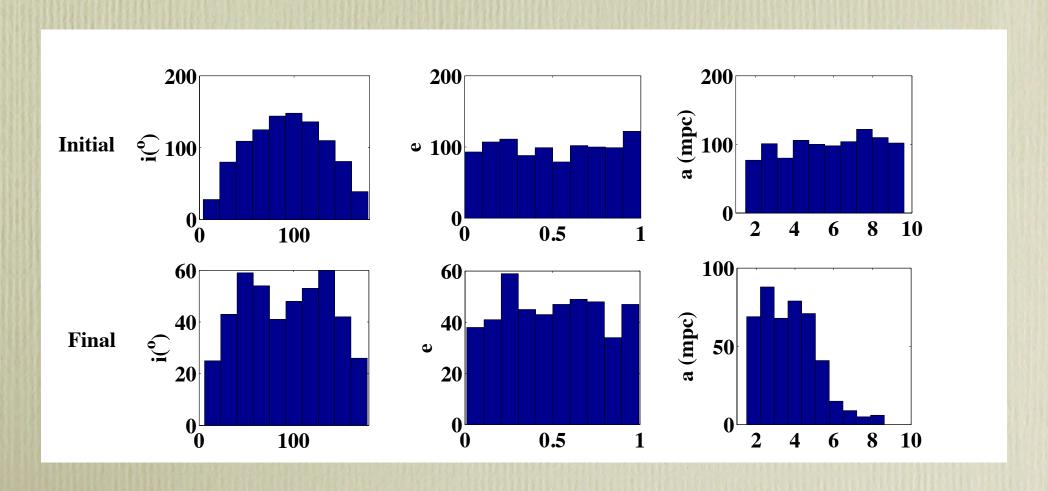
COPLANAR HIGH ECCENTRICITY MIGRATION



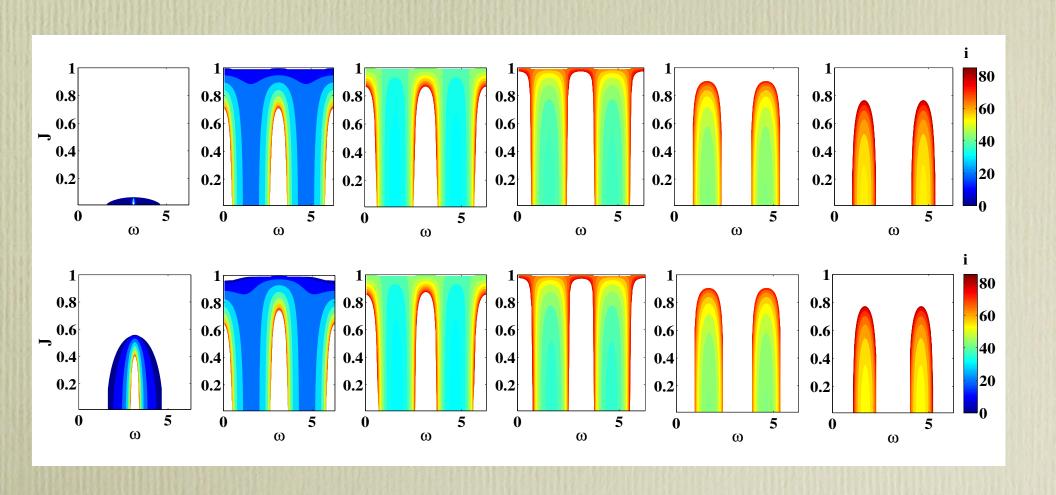
Population synthesis study. tv=0.1yr

Initial v.s. Final Distribution

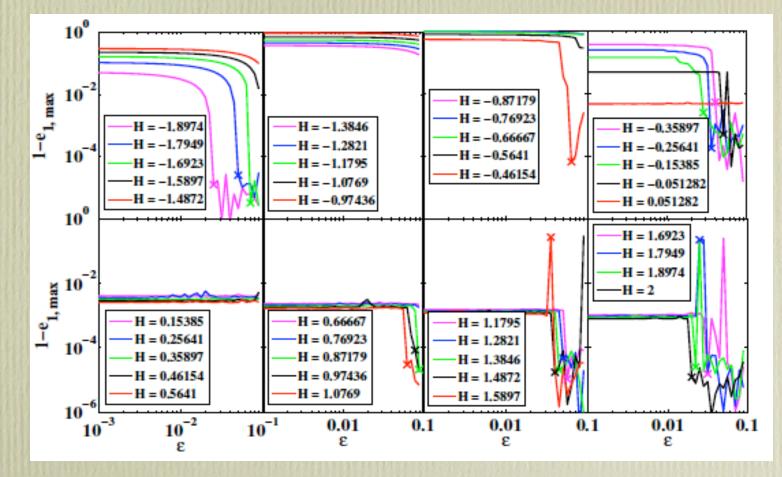
• Example: $m_1 = 10^6 \,\mathrm{M}_{\odot}$, $m_2 = 10^{10} \,\mathrm{M}_{\odot}$, $a_2 = 1 \,\mathrm{pc}$, $e_2 = 0.7$, $\alpha = 1.75$ (stellar distribution), normalized by M- σ relation. Run time: 1Gyr.



Initial Condition in i

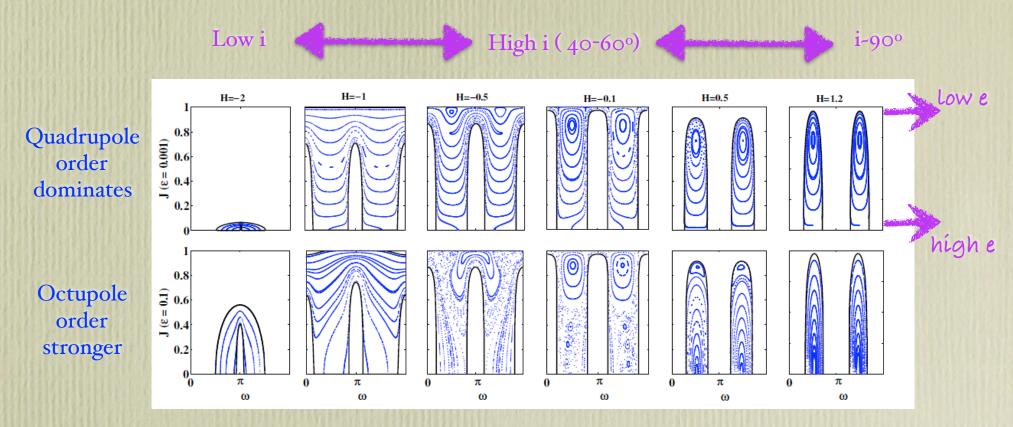


Maximum e₁ for different H and €



Maximum e₁ for low i, high e₁ case, and high i cases

Surface of Section



- Trajectories chaotic only for H=-0.5, -0.1 at high ϵ .
- · High inclination flips are chaotic.
- Overall evolution of the trajectories: evolution sensitive on the initial angles.

Surface of Section

• Surface of section in the $Jz - \Omega$ plane $Jz = \sqrt{1 - e_1^2} \cos i_1 \Omega$: longitude of node

Low i, high e₁ High i, low e₁ H=-2H = -0.5H = -0.3H = -0.1 $Jz (\epsilon = 0.001)$ $\mathbf{Jz} \ (\varepsilon = 0.1)$ <u></u> π π π Ω Ω Ω Ω Ω

Octupole order dominates

Quadrupol

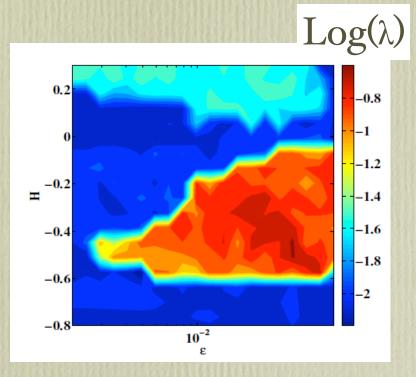
e order

dominates

- All features are due to octupole effects.
- Trajectories are chaotic only possible when H=-0.5, -0.3, -0.1, for high €.

Characterization of Chaos

Lyapunov exponents (λ): λ↑, more chaotic.



- Chaotic when H≤o (correspond to high i cases).
- In chaotic region, Lyapunov timescale $t_L = (1/\lambda) \approx 6t_K$. (t_K corresponds to the oscillation timescale of e_1 and i)

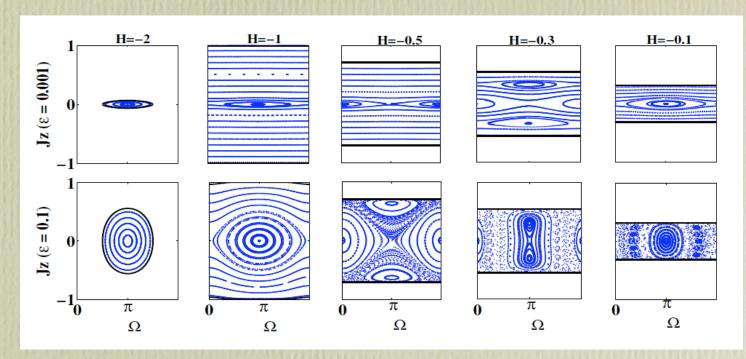
$$t_K = \frac{8}{3} P_{in} \frac{m_1}{m_2} \left(\frac{a_2}{a_1}\right)^3 (1 - e_2^2)^{3/2}$$

Surface of Section

Low i, high e₁ High i, low e₁

Quadrupol e order dominates

Octupole order dominates



- All features are due to octupole effects.
- Trajectories are chaotic only when H≤0.
- Flips are due to octupole resonances.

(Li, et al., 2014 in prep)

Applications --- 2. Tidal Disruption of Stars Surrounding BBH

• SMBHBs originate from mergers between galaxies. Following the merger, the distance of the SMBHB decreases.

(Complete numerical simulations: e.g. Khan et al. 2012)

• SMBHBs with -kpc separation have been observed with direct imagine.

(e.g. Fabbiano et al. 2011, Green et al. 2010, Civano et al. 2010, Komossa et al. 2003, Hutchings & Neff 1989)

• At -1pc separation it is more difficult to identify SMBHBs. SMBHBs have been observed with optical spectra, light variability and radio lines.

(e.g. Boroson & Lauer 2009, Valtonen et al. 2008, Rodriguez et al. 2006)

• Motivation of tidal disruption of stars by -1pc SMBHB: Identify SMBHB at -1 pc separation with tidal disruption rate

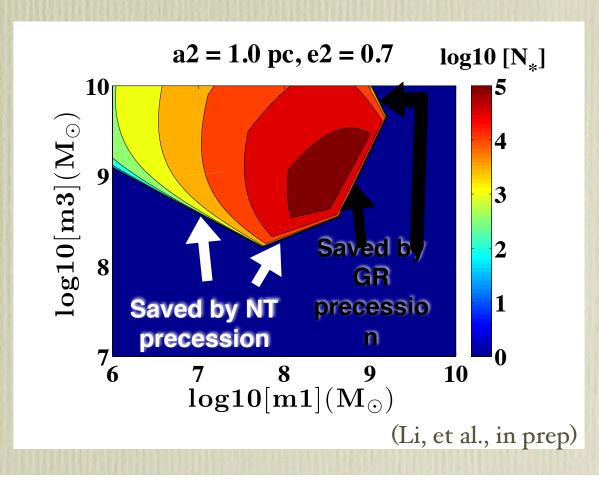
Effects on Stars Surrounding BBH

- Dynamics of stars around BH or BBH:
 - Secular dynamics introduce instability in eccentric stellar disks around a single BH (e.g. *Madigan*, *Levin & Hopman* 2009)
 - Tidal disruption event rate can be enhanced due to BBH and the recoil of BBH (*Ivanov et al. 2005, Wegg & Bode 2011, Chen et al. 2011, Stone & Loeb 2011*)
 - Relic stellar clusters of recoiled BH may uncover MW formation history (e.g. O'Leary & Loeb 2009).
- Here we study the effect of EKM to stars surrounding BBH

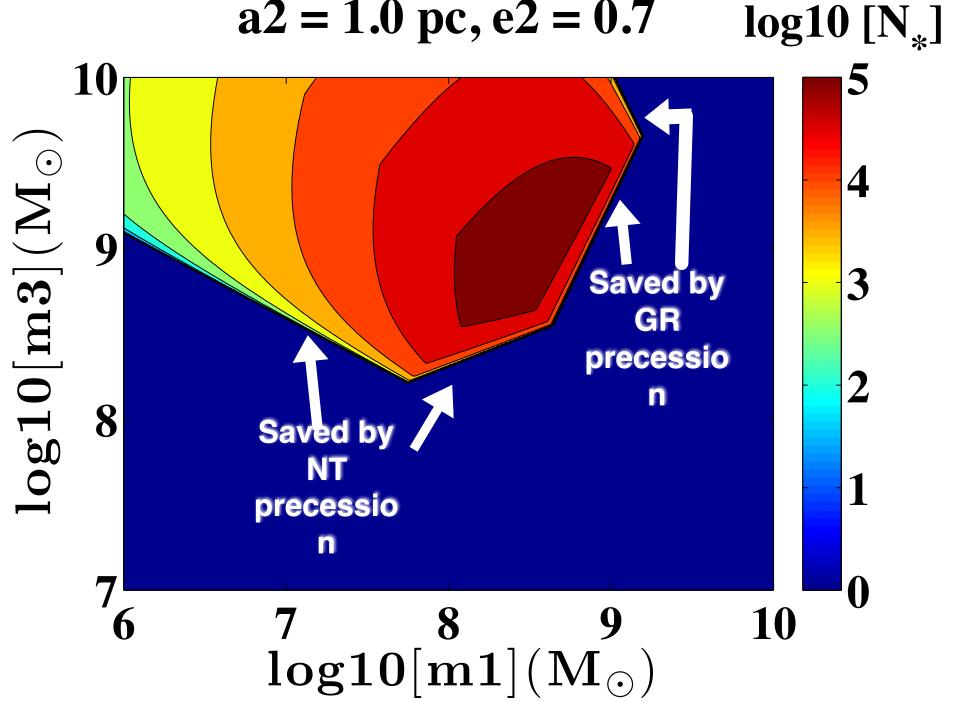
Effects of EKM on Stars Surrounding BBH

- Study the role of eccentric (e₂ ≠ 0) Kozai mechanism in the presence of general relativistic (GR) precession and Newtonian (NT) precession for stars surrounding SMBHB.
- Set the separation of the BBH at $a_2=1pc$, $e_2=0.7$ and assuming $o_* \propto a^{-1.75}$, normalized by M- σ relation.
- N* is the number of stars affected by the eccentric Kozai Mechanism.

 (Requirement: t_{GR} < t_{Kozai}, t_{NT} < t_{Kozai}, ε < 0.1, α₁ < r_{RL}).

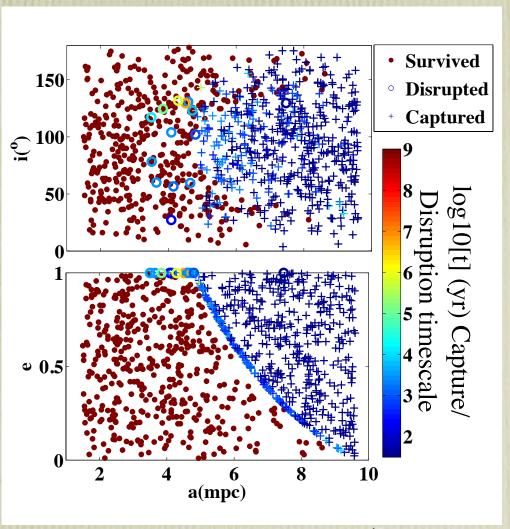


a2 = 1.0 pc, e2 = 0.7

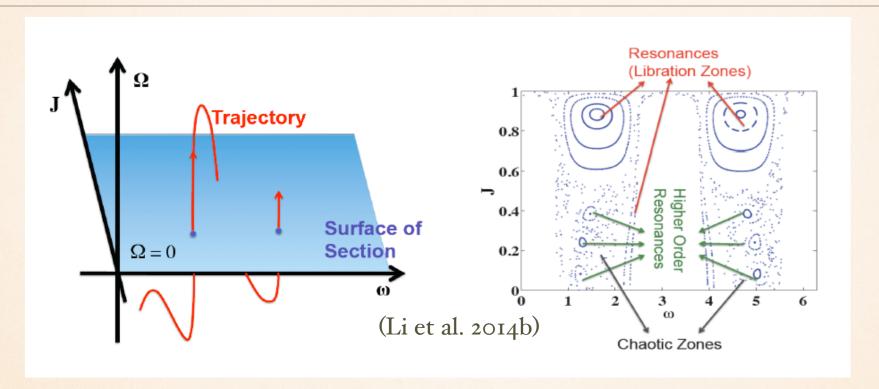


Effects of EKM on Stars Surrounding BBH

- Example: $m_1 = 10^6 \,\mathrm{M}_{\odot}$, $m_2 = 10^{10} \,\mathrm{M}_{\odot}$, $a_2 = 1 \,\mathrm{pc}$, $e_2 = 0.7$, Run time: 1Gyr.
- 14/1000 disrupted; 535/1000 captured. Disruption/capture timescales are short.
 - => Captured stars may change stellar density profile of the other BH
 - => With rapid diffusion, disruption rate ~10-3/yr.

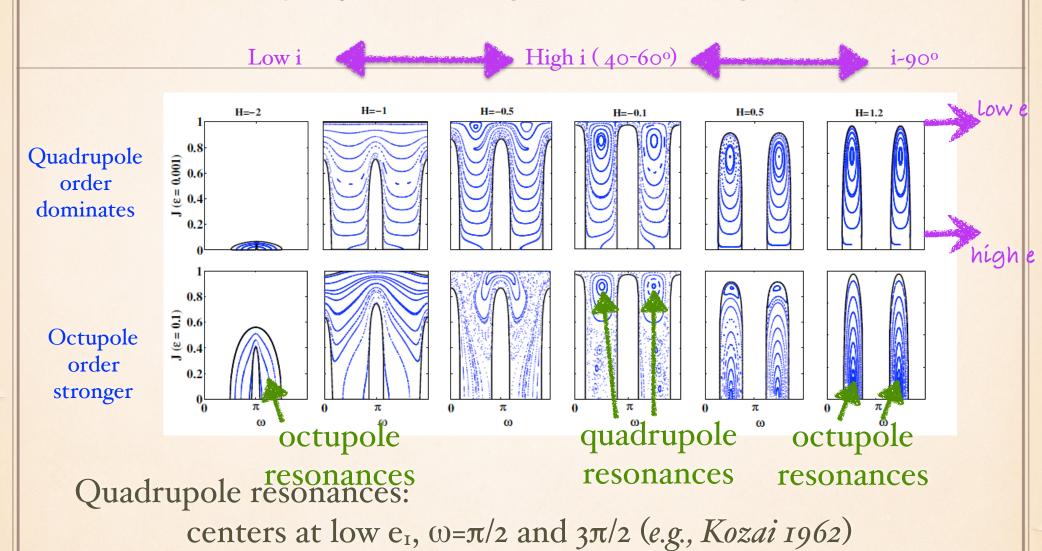


SURFACE OF SECTION



- Resonant zones: points fill 1-D lines. trajectories are quasi-periodic.
- Chaotic zones: points fill a higher dimension.
 trajectories are chaotic.

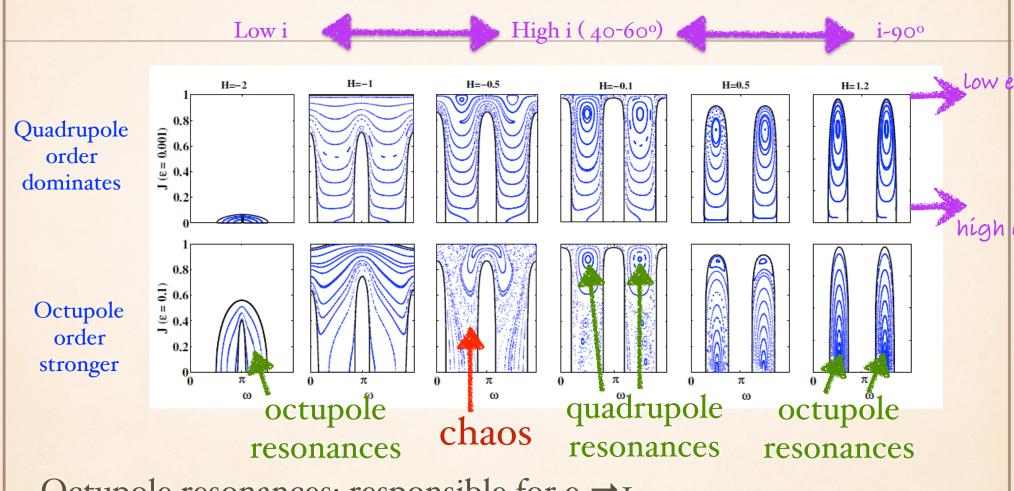
SURFACE OF SECTION



centers at high e_1 , $\omega = \pi$ or $\pi/2$ and $3\pi/2$

Octupole resonances:

SURFACE OF SECTION

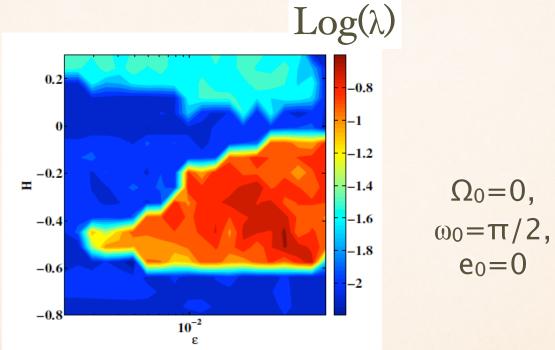


Octupole resonances: responsible for e →1

Chaos: overlap of quadrupole and octupole resonances high inclination flips

CHARACTERIZATION OF CHAOS

Chaotic when H≤o (correspond to high i cases).

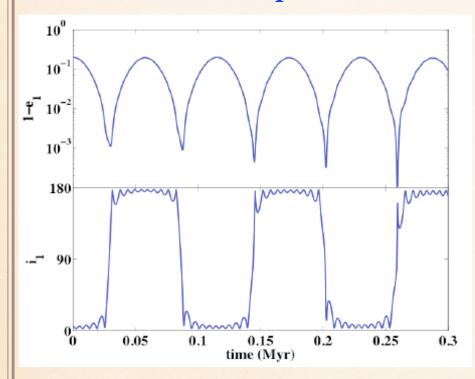


In chaotic region, Lyapunov timescale t_L=(1/λ) ≈ 6t_K.
 (t_K corresponds to the oscillation timescale of e₁ and i)

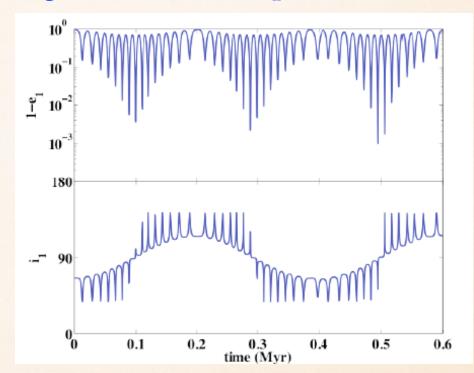
$$t_K = \frac{8}{3} P_{in} \frac{m_1}{m_2} \left(\frac{a_2}{a_1}\right)^3 (1 - e_2^2)^{3/2}$$

DIFFERENCES BETWEEN HIGH/LOW I FLIP

Low inclination flip



High inclination flip

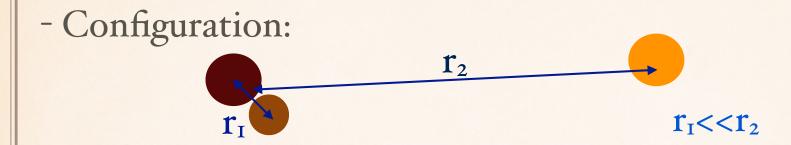


Low inclination flips:

- e_{t} monotonically, inclination stays low before flip.
- i stays low before flip.

(Li et al. 2014a)

HIERARCHICAL THREE-BODY SYSTEMS



- Hierarchical configurations are COMMON:
- For binaries with periods shorter than 10 days, >40% of them are in systems with multiplicity ≥ 3. (Tokovinin 1997)
- For binaries with period < 3 days, ≥96% are in systems with multiplicity ≥3. (*Tokovinin et al. 2006*)
- 282 of the 299 triple systems (- 94.3%) are hierarchical. (Eggleton et al. 2007)
- Hierarchical 3-body dynamics gives insight for hierarchical multiple systems.

EXAMPLES OF HIERARCHICAL 3-BODY DYNAMICS

For stellar systems:

Short Period Binaries

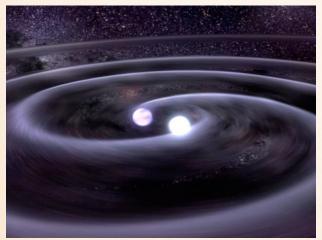
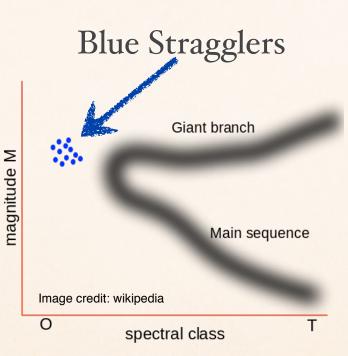


Image credit: NASA/Tod Strohmayer/Dana Berry

e.g., Harrington 1969; Mazeh & Shaham 1979; Ford et al. 2000; Eggleton & Kiseleva-Eggleton 2001; Fabrycky & Tremaine 2007; Shappee & Thompson 2013



e.g., Perets & Fabrycky 2009; Naoz & Fabrycky 2014

Type Ia Supernova

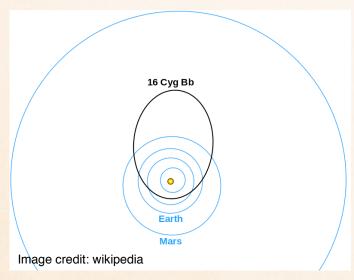


e.g., Katz & Dong 2012; Kushnir et al. 2013

EXAMPLES OF HIERARCHICAL 3-BODY DYNAMICS

Exoplanetary systems:

Eccentric Orbits



e.g., Holman et al. 1997; Ford et al. 2000; Wu & Murray 2003;

Exoplanets with large spinorbit misalignment

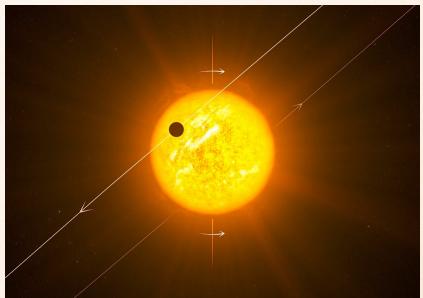


Image credit: ESO/A. C. Cameron

e.g., Fabrycky & Tremaine 2007; Naoz et al. 2011, 2012; Petrovich 2015; Storch et al. 2014; Anderson et al. 2016

EXAMPLES OF HIERARCHICAL 3-BODY DYNAMICS

Black hole systems:

Merger of short period black hole binaries

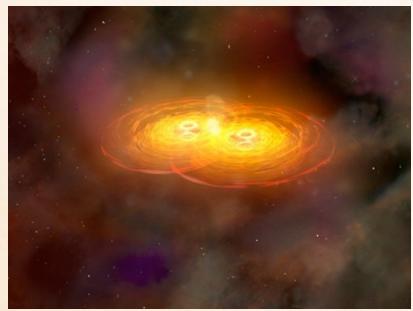


Image credit: NASA / CXC / A. Hobart

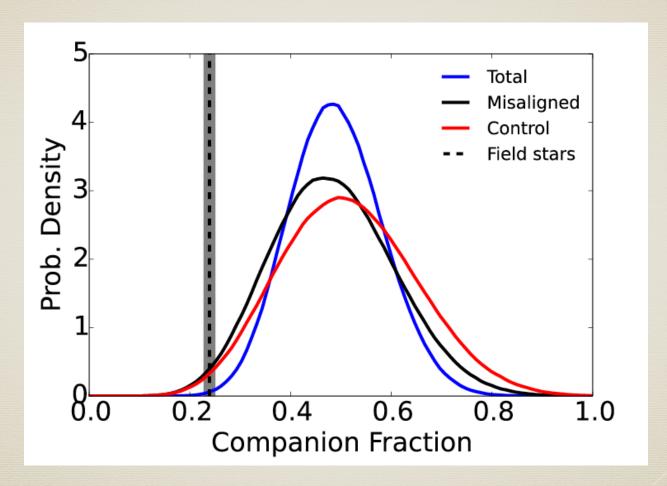
e.g., Blaes et al. 2002; Miller & Hamilton 2002; Wen 2003; Bode & Wegg 2014; Tidal disruption events



Image credit: NASA/CXC/M.Weiss

e.g., Chen et al. 2009, 2011; Wegg & Bode 2011; Li et al. 2015

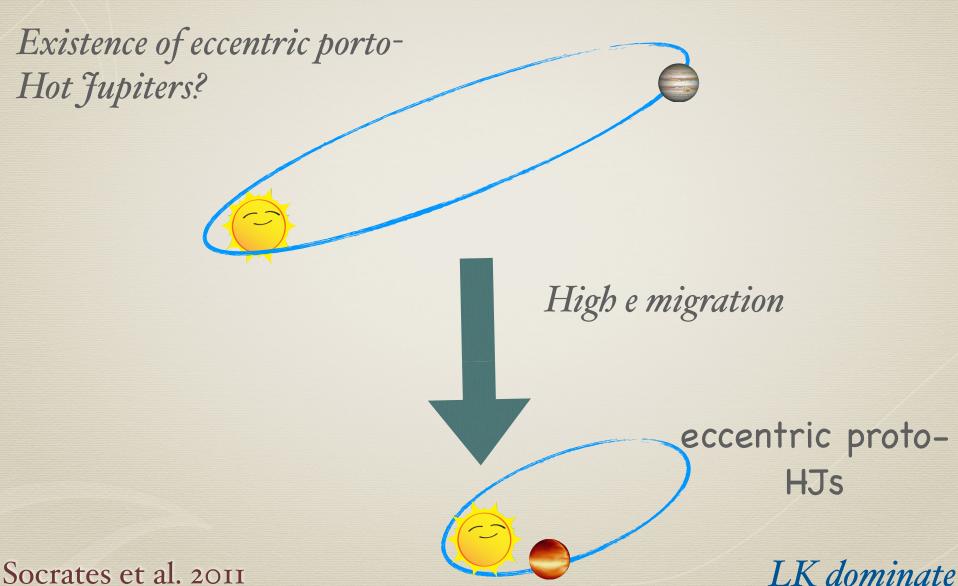
Spin-orbit Misalignment



* No correlation between misaligned/eccentric hot Jupiter systems and the incidence of stellar companions

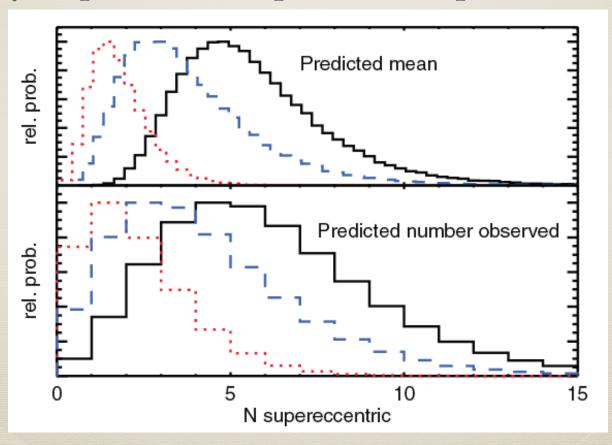
Ngo et al. 2015

Eccentric Proto-Hot Jupiters



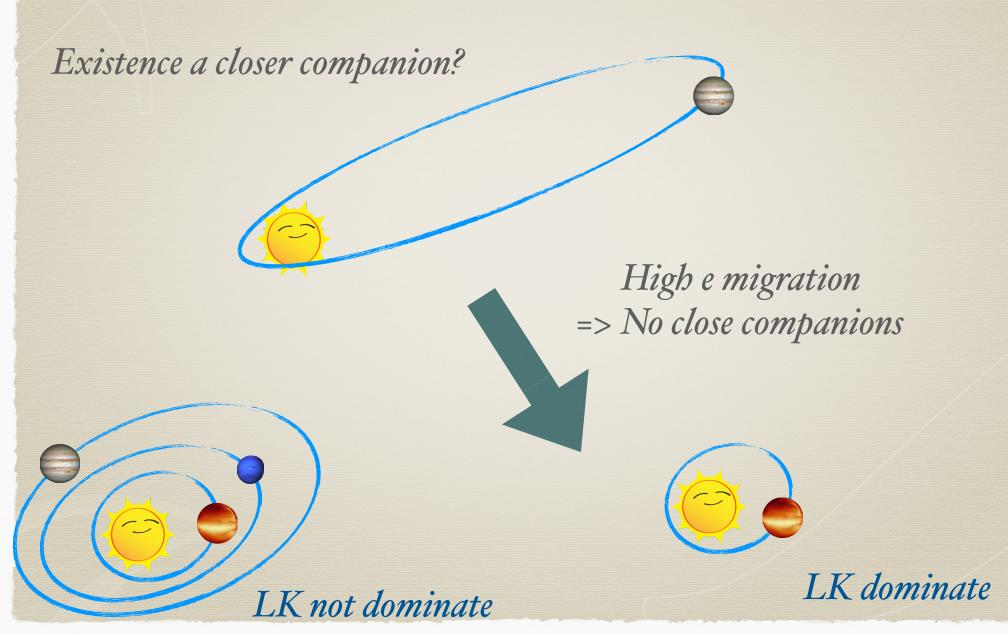
Proto-Hot Jupiters

* A paucity of proto-hot Jupiters on super-eccentric orbits

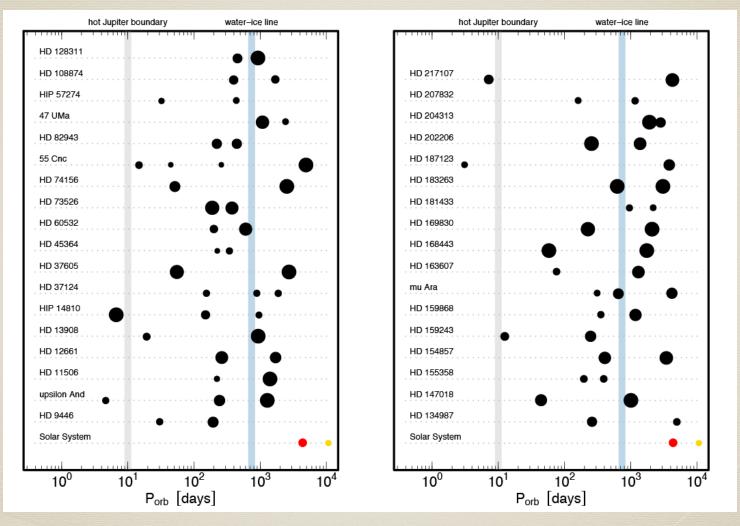


* <44% formed via LK mechanism

Closer Companions of Hot Jupiters



Closer Companions of Hot Jupiters



Hot Jupiters (< 10 days) are no more or less likely to have exterior companions than giant planets (>10 days)

=> high e migration does not dominate

Schlaufman & Winn 2016